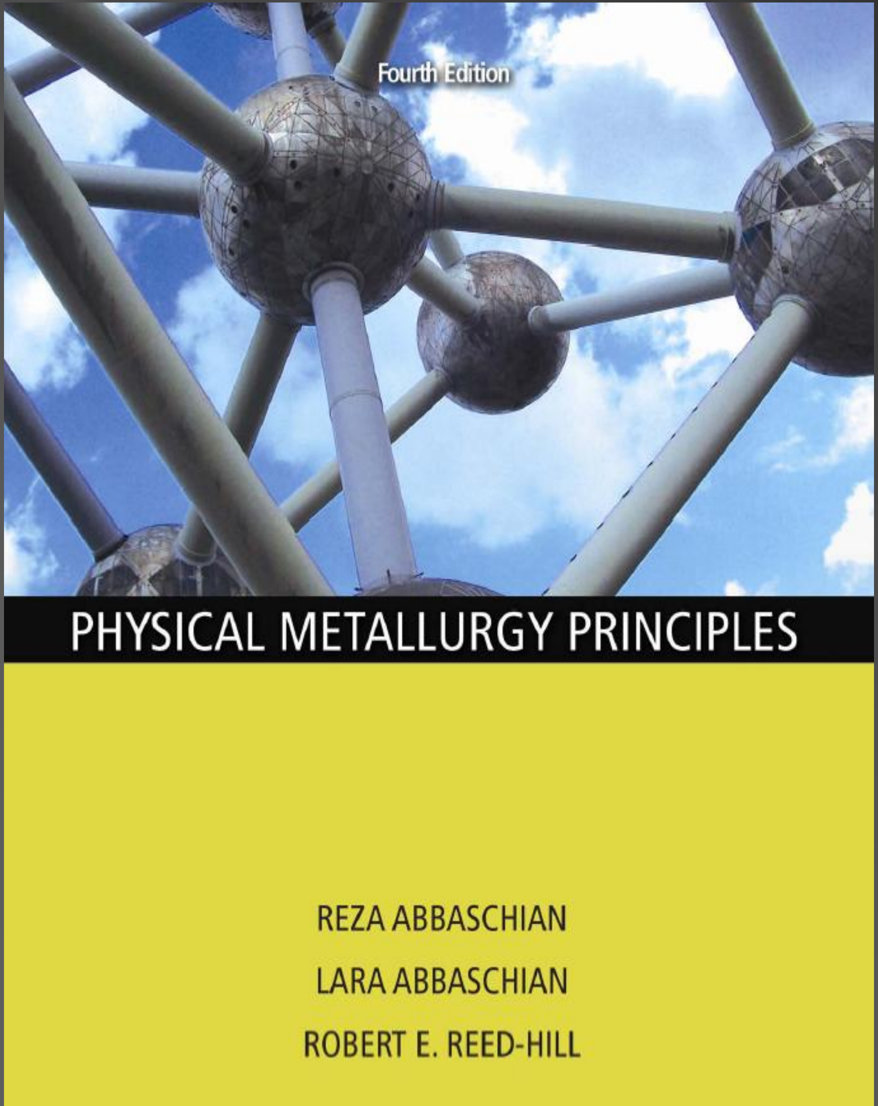
Week	Tue.	Fri.	Note
1.	2/24 (Chap. 1)	**2/27 (Holiday)**	**Chap. 1: 2 h**
2.	3/3 (Chap. 1)	3/6 (Chap. 3)	**Chap. 3: 2 h**
3.	3/10 (Chap. 2)	3/13 (Chap. 2)	**Chap. 2: 6 h**
4.	3/17 (Chap. 2)	3/20 (Chap. 2)	
5.	3/24 (Chap. 4)	**3/27 (1st exam)**	**1st exam: Chap. 1-3**
6.	3/31 (Chap. 4)	**4/3 (Spring break)**	**Chap. 4: 6 h**
7.	**4/7 (Spring break)**	4/10 (Chap. 4)	
8.	4/14 (Chap. 4)	4/17 (Chap. 4, 5)	
9.	4/21 (Chap. 5)	4/24 (Chap. 5)	**Chap. 5: 6 h**
10.	4/28 (Chap. 5)	**5/1 (Holiday)**	
11.	5/5 (Chap. 5)	5/8 (Chap. 6)	**Chap. 6: 6 h**
12.	5/12 (Chap. 6)	5/15 (Chap. 6)	**Chap. 7: 2 h**
13.	5/19 (Chap. 6)	5/22 (Chap. 7)	**Defects: 1 h**
14.	5/26 (Defects)	**5/29 (2nd exam)**	**2rd exam: Chap.4-6**
15.	6/2 (Chap. 8)	6/5 (Chap. 8)	
16.	6/9 (Chap. 8)	6/12 (Chap. 8)	**Chap. 8: 7 h**
17.	6/16 (Chap. 8)	**6/19 (Holiday)**	
18.	**6/23 (no class)**	**6/26 (Final exam)**	**Final: Defects & Chap. 7-8**

Note: 6/22-26: final exam week.

PHYSICAL METALLURGY PRINCIPLES

Fourth Edition

SI Version



Fourth Edition

PHYSICAL METALLURGY PRINCIPLES

REZA ABBASCHIAN

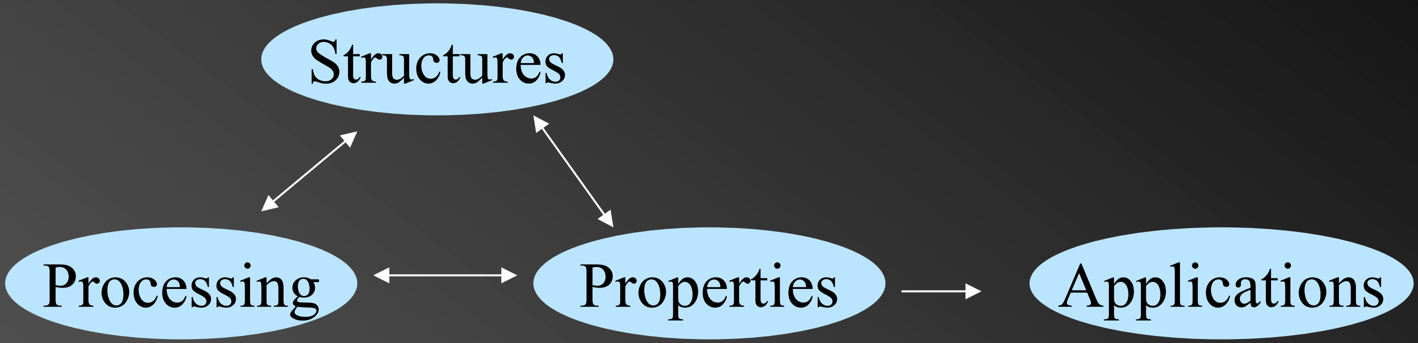
LARA ABBASCHIAN

ROBERT E. REED-HILL

Chapter One:

The Structure of Metals

1.1 Importance of the structure:



Structures

Processing

Properties

Applications

▲ Classification for the Structures:

(1) Macrostructure: can be easily examined by naked eyes, or low magnification methods

(2) Microstructure: need high magnification methods to examine; e.g. grains, phases, precipitations, defects, etc.

(3) Crystal structure: different orientation => polycrystalline

(4) The size range of the “Structure” is defined from macroscopic arrangements of the different materials in engineering objects (like airplane, automobile, bridge, etc) to atomic scale arrangements of the molecules and atoms (like dislocations, surface kinks, vacancies, different crystals structures, etc.

1.2 Metallographic Specimen Preparation

- No absolute ways of preparations
- Variations: depending on the nature of the metal to be examined.

  Cutting => Mounting (cold or hot mounting) =>
  Grinding (sand papers with different grit size) =>
  Rough polishing (paste or slurry of Al_2O_3 or diamond with different particle size was used) =>

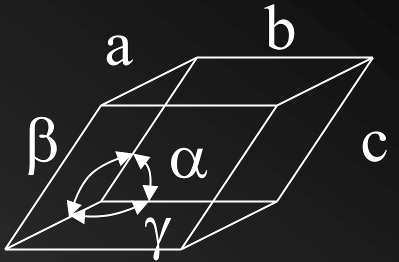
Final polishing (Mechanical polishing: paste or slurry with even smaller particle size was used;
Chemical polishing; Electro-polishing;
Chemical-mechanical polishing) =>
Etching (simple chemical or Electro-etching) =>
Observations

- Etching process:
  to produce height difference between defects, phases, grains with different orientations for observations in the optical or electron microscope.
  Etching involves alternate processes of oxidation and dissolution. General places to look for etching solutions: the appendix of TEM textbooks.

1.3 Crystal Structure of metals:

Crystal: an orderly array of atoms in space.

7 kinds of lattice with total 14 variations (Bravais lattice): unit cel



(1) Cubic => $a = b = c$, $\alpha = \beta = \gamma = 90^o$;
3: simple cubic, body-centered cubic, face-centered cubic.

(2) Tetragonal=> $a = b \neq c$, $\alpha = \beta = \gamma = 90^o$;
2: simple tetragonal, body-centered tetragonal.

(3) Orthorhombic => $a \neq b \neq c$, $\alpha = \beta = \gamma = 90^o$;
4: simple orthorhombic, based-centered, body-centered orthorhombic, face-centered orthorhombic.

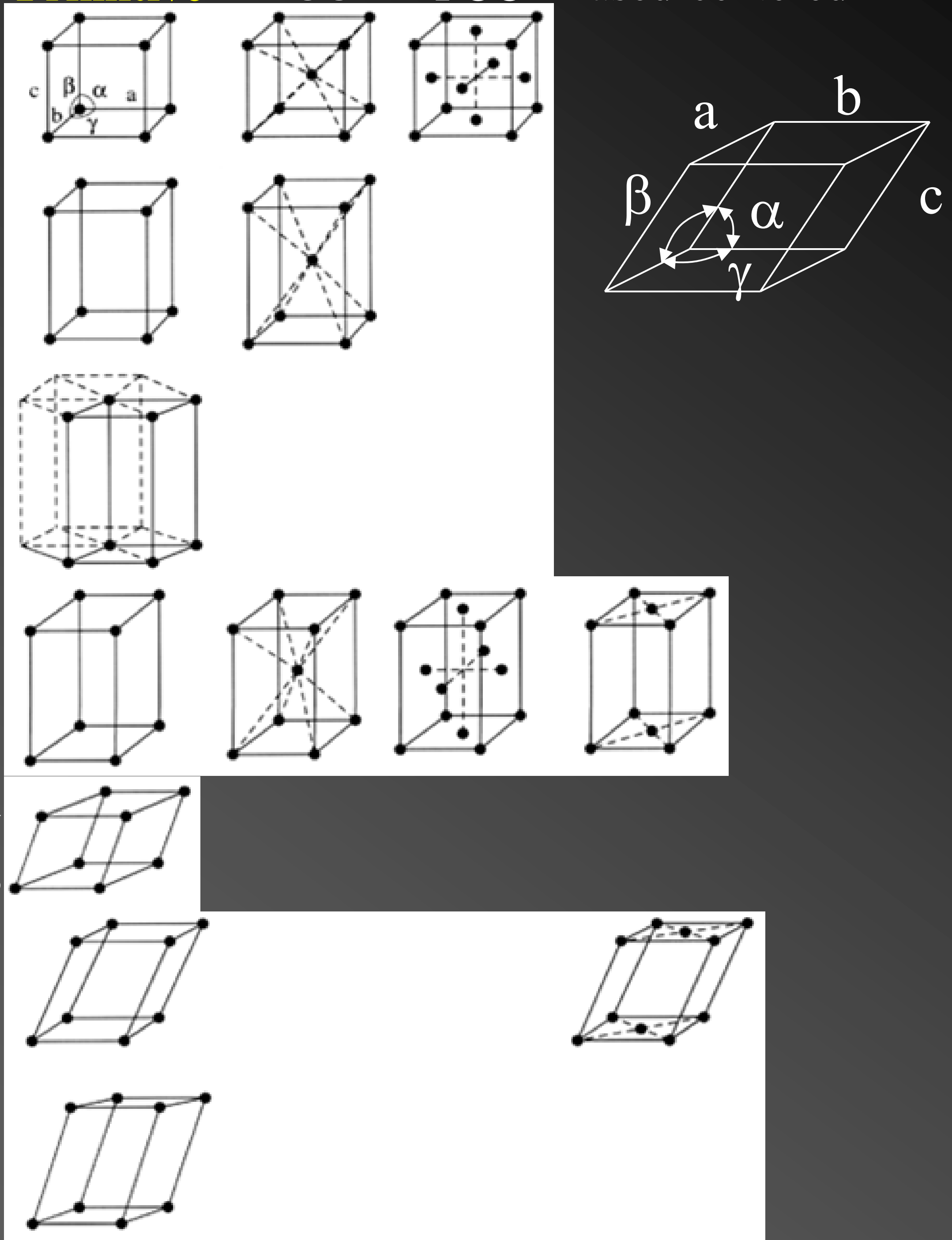
(4) Monoclinic => $a \neq b \neq c$, $\alpha = \gamma = 90^o \neq \beta$;
2: simple monoclinic, base-centered monoclinic.

(5) Triclinic => $a \neq b \neq c$, $\alpha \neq \beta \neq \gamma$

(6) Trigonal (Rhombohedral) => $a = b = c$,
$\alpha = \beta = \gamma \neq 90^o$ & $<120^o$

(7) Hexagonal => $a = b \neq c$, $\alpha = \beta = 90^o$, $\gamma = 120^o$

Primitive BCC FCC Based-centered



1. Cubic

2. Tetragonal

$a = b \neq c$

$\alpha = \beta = \gamma = 90^o$

3. Hexagonal

$a = b \neq c$

$\alpha = \beta = 90^o$

$\gamma = 120^o$

4. Orthorhombic

$a \neq b \neq c$,

$\alpha = \beta = \gamma = 90^o$

5. Rhombohedral

$a = b = c$, (Trigonal)

$\alpha = \beta = \gamma \neq 90^o$ & $<120^o$

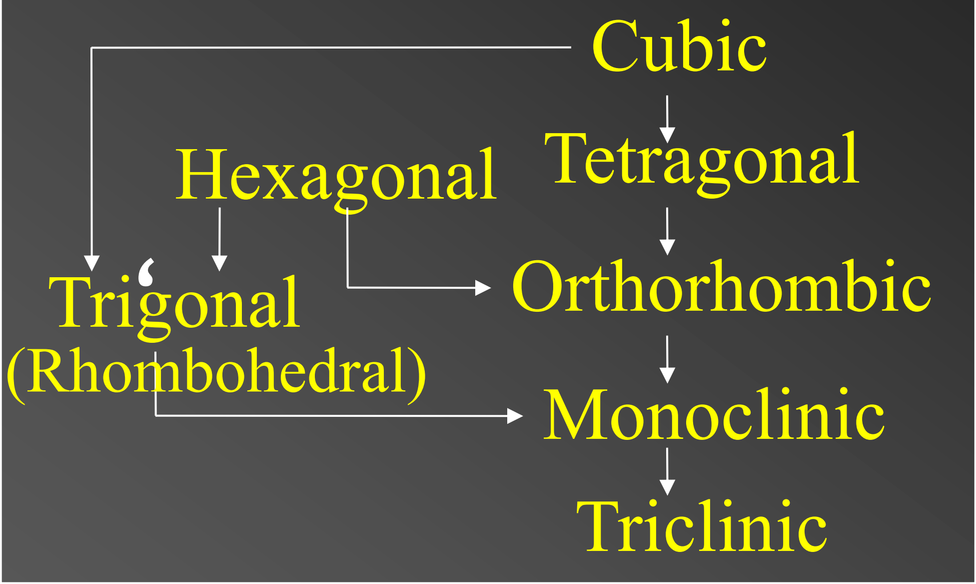
6. Monoclinic

$a \neq b \neq c$

$\alpha = \gamma = 90^o \neq \beta$

7. Triclinic

Symmetric hierarchy



Cubic

Hexagonal

Tetragonal

Trigonal
(Rhombohedral)

Orthorhombic

Monoclinic

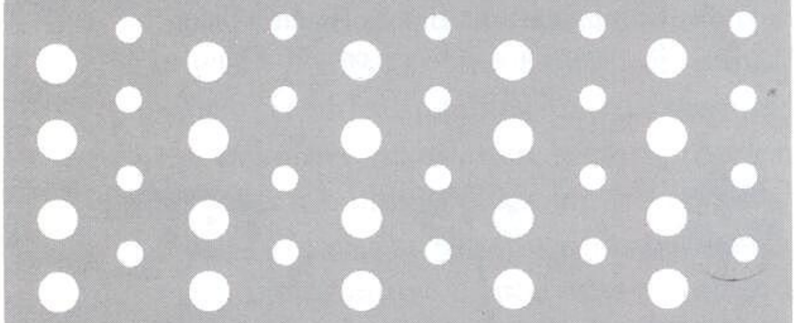
Triclinic

From N.W. Ashcroft and N.D. Mermin, Solid State Physics., 1976.

- Crystal Structure ≡ Space Lattice (Bravais) + Basis
  - A crystal is made up of a periodic arrangement of one or more atoms (the *basis*) repeated at each lattice point.

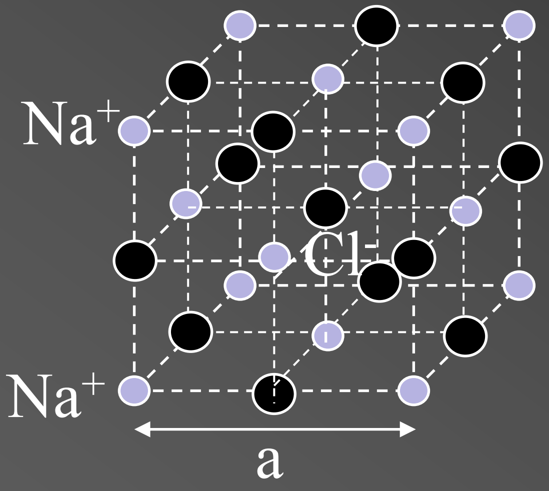
(a) Space lattice

(b) Basis, containing two different ions



(c) Crystal structure

NaCl: CN=6

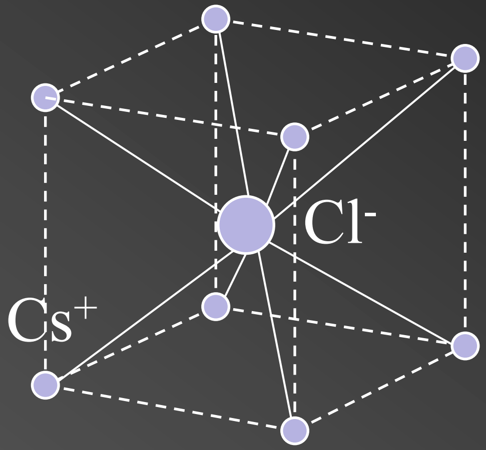


Two interpenetrating FCC

FCC Bravais with a basis of

Na: 0, Cl: $(a/2)(\hat{x} + \hat{y} + \hat{z})$

CsCl: CN=8



Simple cubic Bravais with a basis of

Cs: 0, Cl: $(a/2)(\hat{x} + \hat{y} + \hat{z})$

Zincblend (two kind of atoms): CN=4

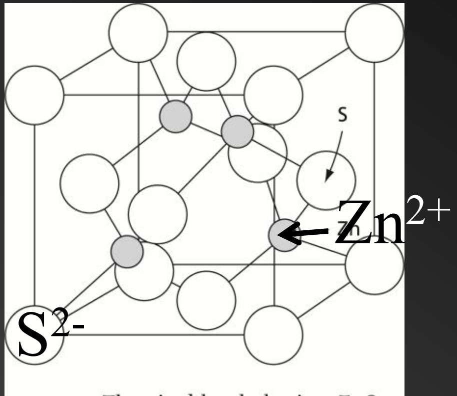


FIG. 3.3 The zincblende lattice, ZnS

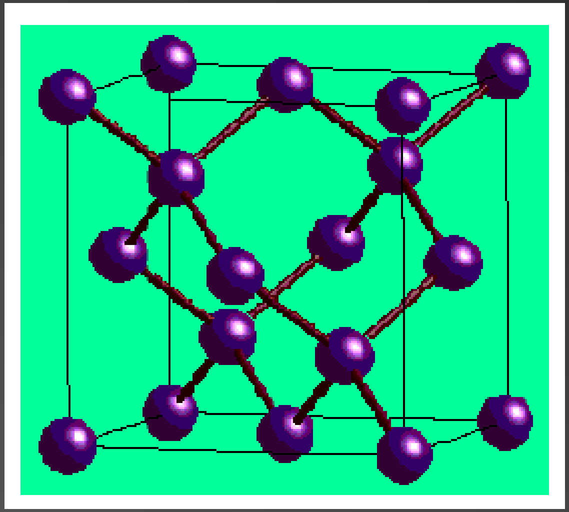
(diamond structure: Not a Bravasis)

FCC with a basis of

S: 0, Zn: $(a/4)(\hat{x} + \hat{y} + \hat{z})$

- Crystal Structure ≡ Space Lattice (Bravais) + Basis

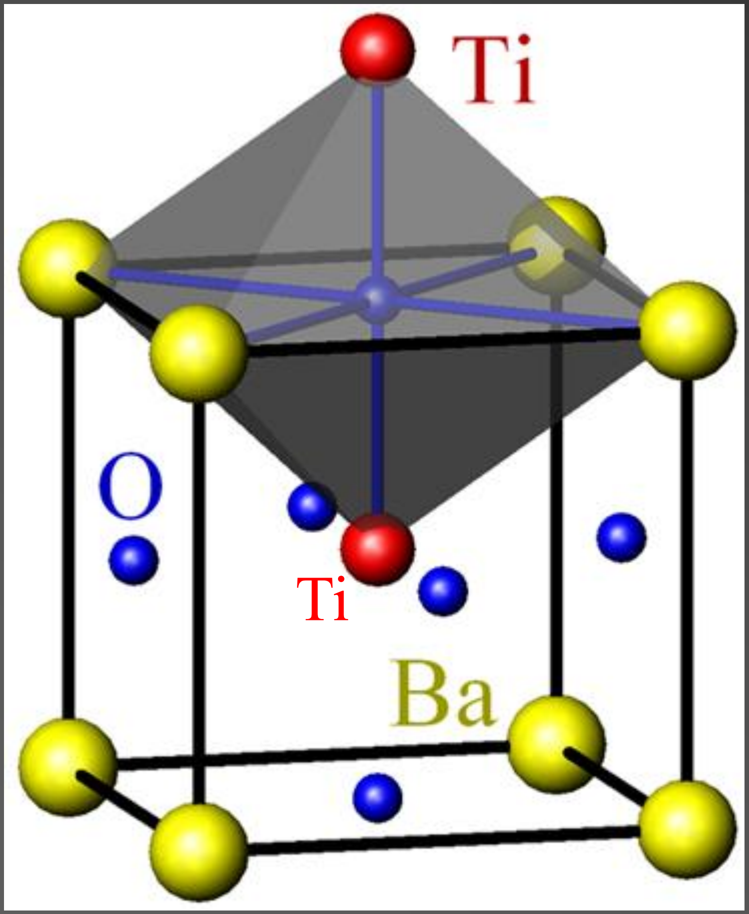
Diamond structure



Lattice = FCC

Basis = C (0,0,0), C (1/4,1/4,1/4)

Perovskite structure: $BaTiO_3$

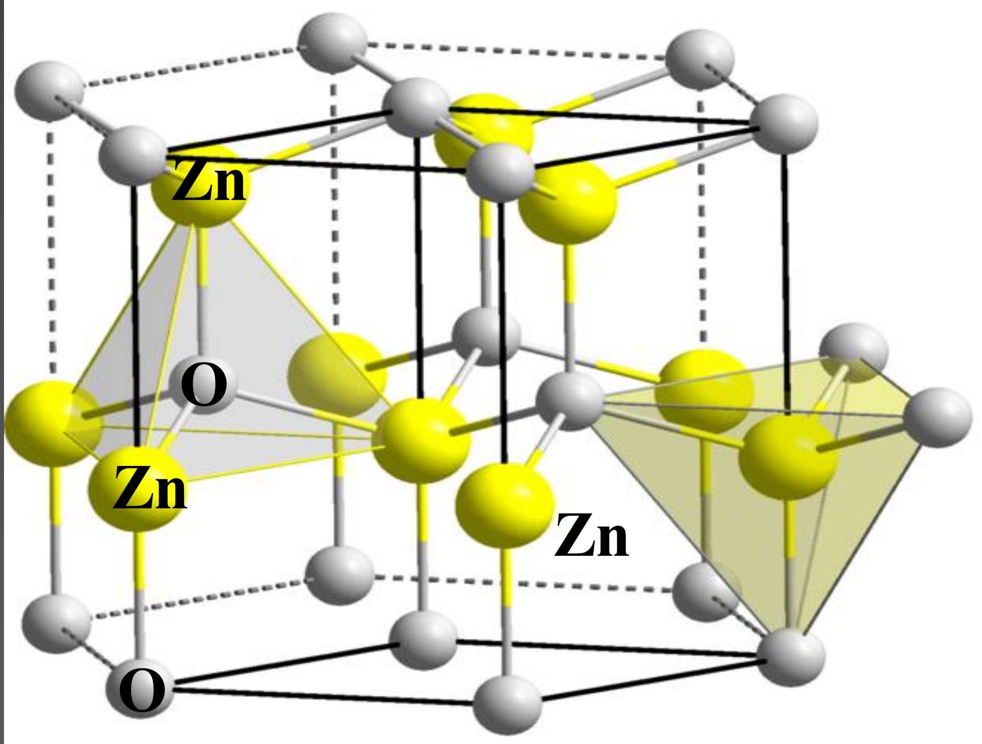


Lattice = Primitive cubic

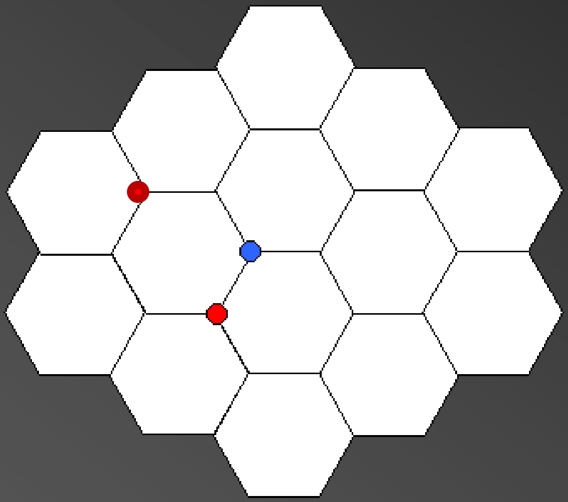
Basis = Ti (0,0,0), Ba (½,½,½),

O (½,0,0), (0, ½,0) & (0,0,½)

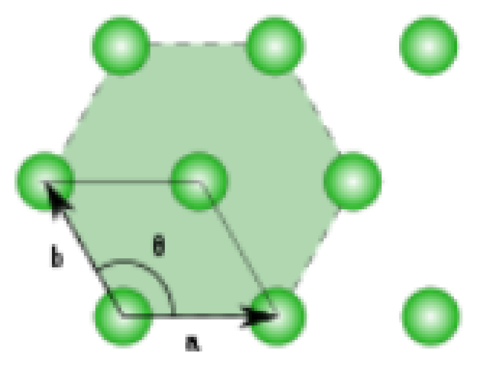
ZnO: Wurtzite structure



- **Lattice** = Primitive hexagonal
- **Basis** = O (0,0,0) & (2/3,1/3,1/2)
  Zn (0,0,5/8) & (2/3,1/3,1/8)



Compared with



Honeycomb Lattice

- The red & blue dots are not equivalent. If the blue side is rotated through 180º the lattice is invariant.

  ⇒ The Honeycomb lattice is NOT a Bravais Lattice.

But, it can be said as

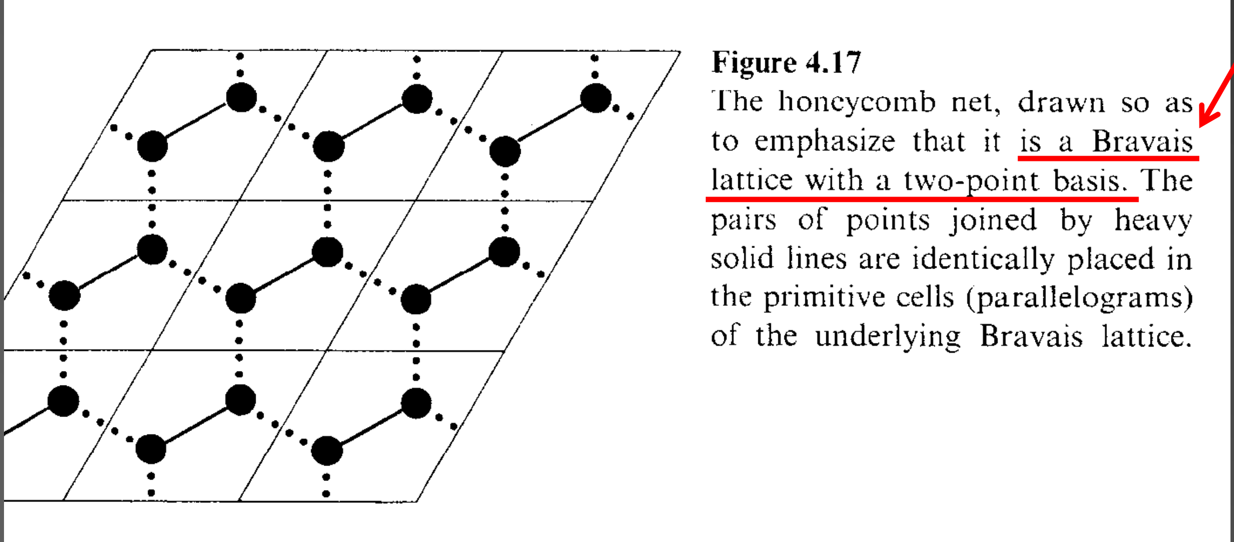
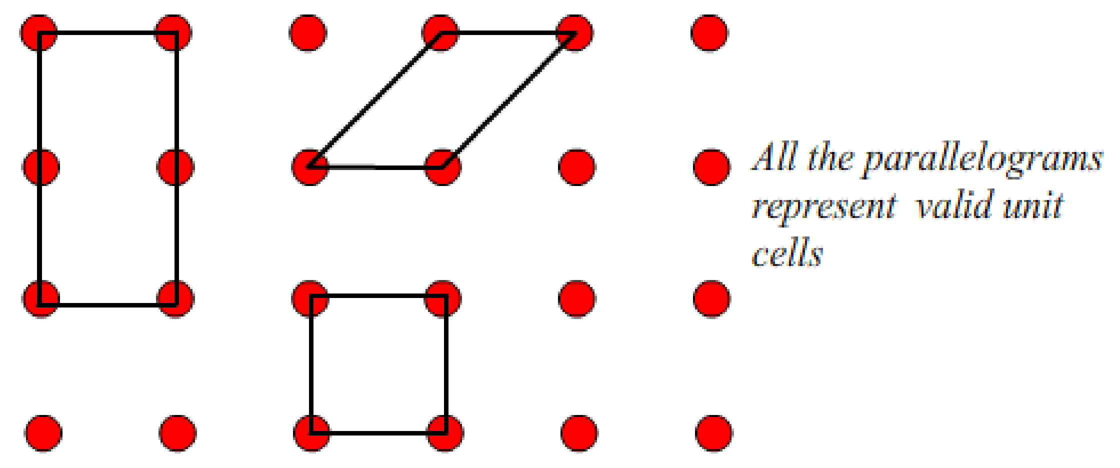


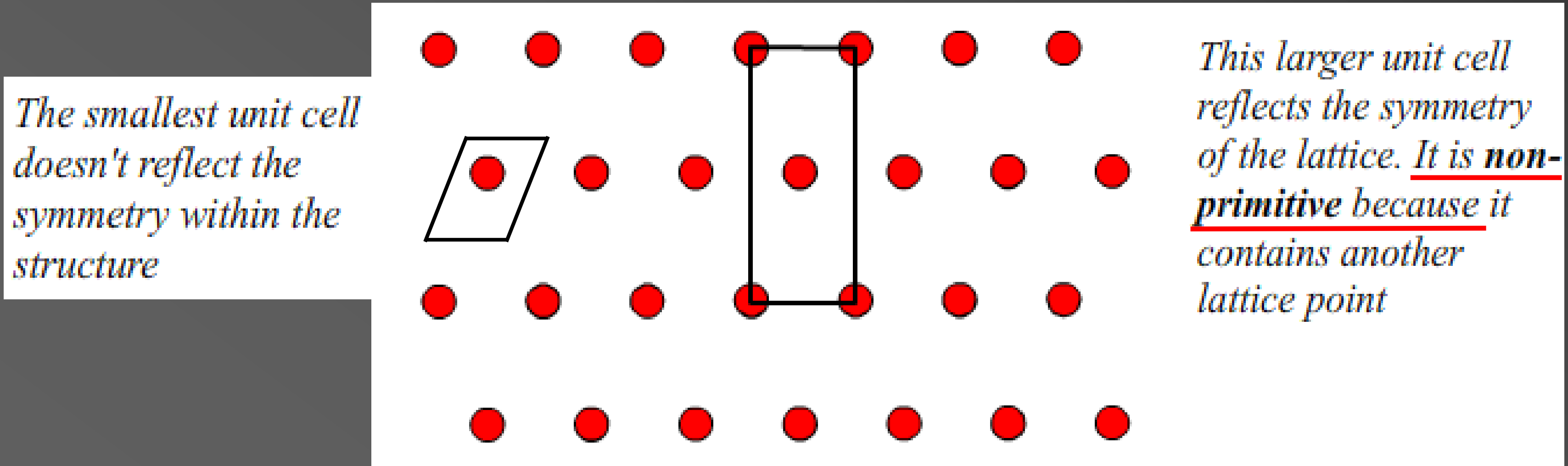
Figure 4.17
The honeycomb net, drawn so as to emphasize that it is a Bravais lattice with a two-point basis. The pairs of points joined by heavy solid lines are identically placed in the primitive cells (parallelograms) of the underlying Bravais lattice.

1.4 Unit Cell: the group of atoms possessing the system of the crystal which, when repeated in all directions, will develop the crystal lattice.



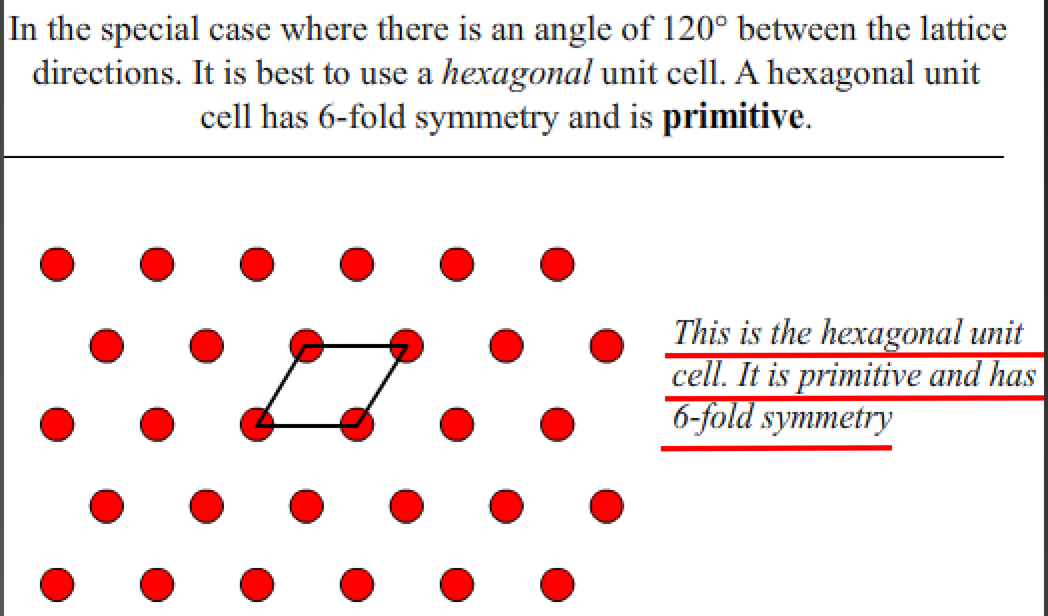
All the parallelograms represent valid unit cells

- In practice, it is best to choose the **smallest** unit cell or the unit cell which provides **the most symmetry**.



The smallest unit cell doesn't reflect the symmetry within the structure

*This larger unit cell reflects the symmetry of the lattice. It is **non-primitive** because it contains another lattice point*



In the special case where there is an angle of 120° between the lattice directions. It is best to use a *hexagonal* unit cell. A hexagonal unit cell has 6-fold symmetry and is **primitive**.

This is the hexagonal unit cell. It is primitive and has 6-fold symmetry

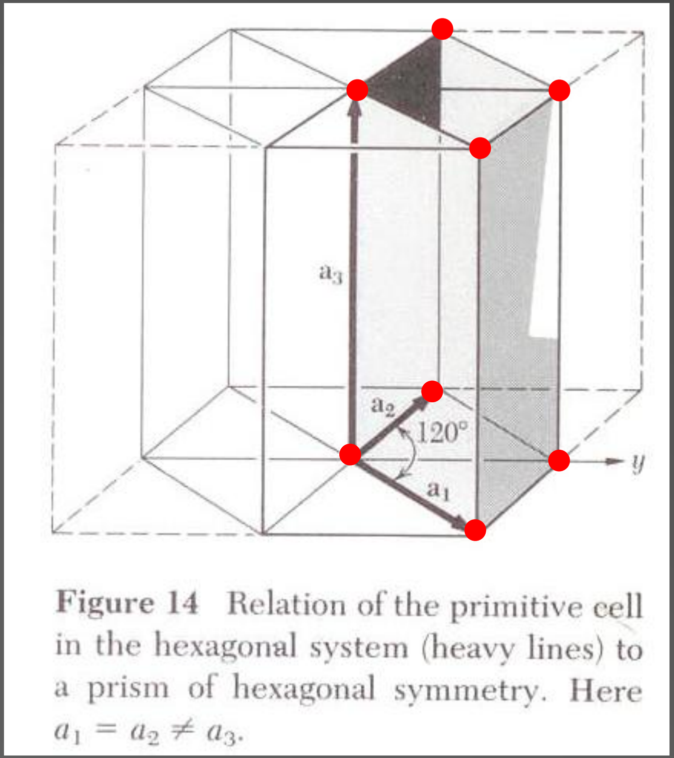
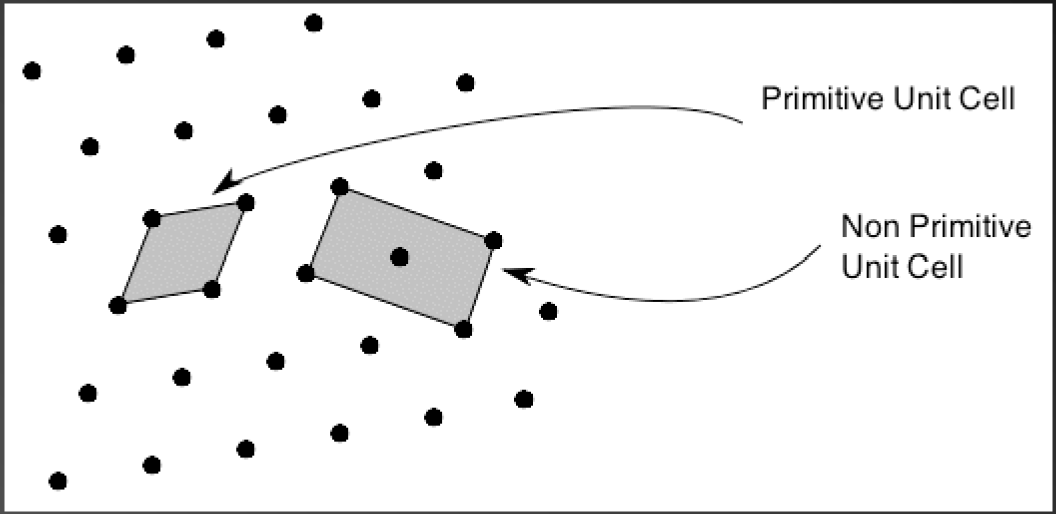


Figure 14 Relation of the primitive cell in the hexagonal system (heavy lines) to a prism of hexagonal symmetry. Here $a_1 = a_2 \neq a_3$.

hexagonal Bravais lattice

Conventional Cell
= Primitive Cell

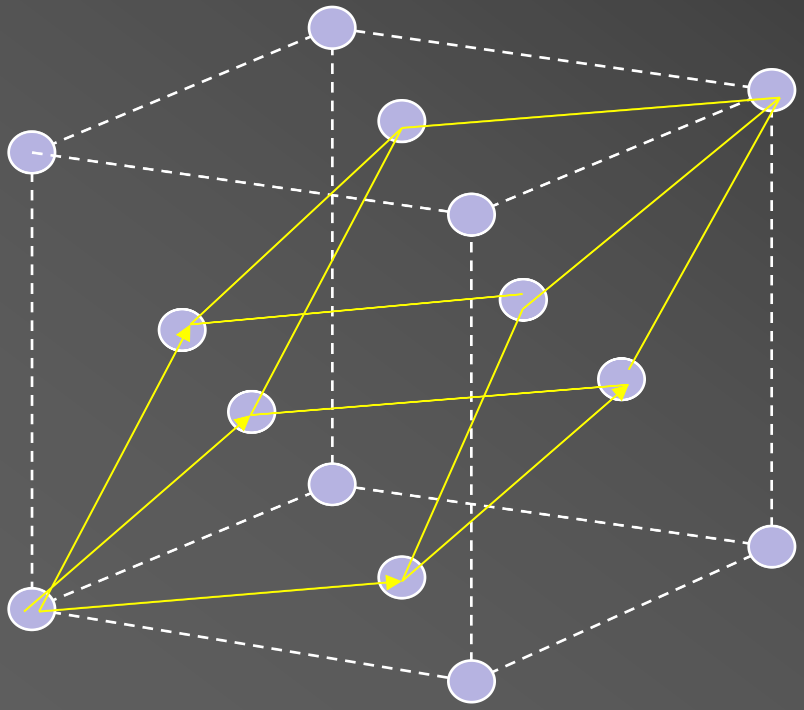


- Primitive unit cells contain **only one lattice point**, which is made up from the lattice points at each of the corners.

- Non-primitive unit cells contain **additional lattice points**, either on a face of the unit cell or within the unit cell, and so have more than one lattice point per unit cell.

http://www.doitpoms.ac.uk/tlplib/crystallography3/unit_cell.php

Primitive unit cell: is a type of minimum-volume cell that will fill all space by the repetition of suitable translation operation.



FCC

a = b = c, (Trigonal)

$\alpha = \beta = \gamma \neq 90^o$ **& <**120^o

— Primitive unit cell(Rhombohedron)

Contain only one lattice point

The smallest unit cell

Primitive basis vectors

- - - - Unit Cell

Basis vectors

Conventional unit cell

BCC

FCC

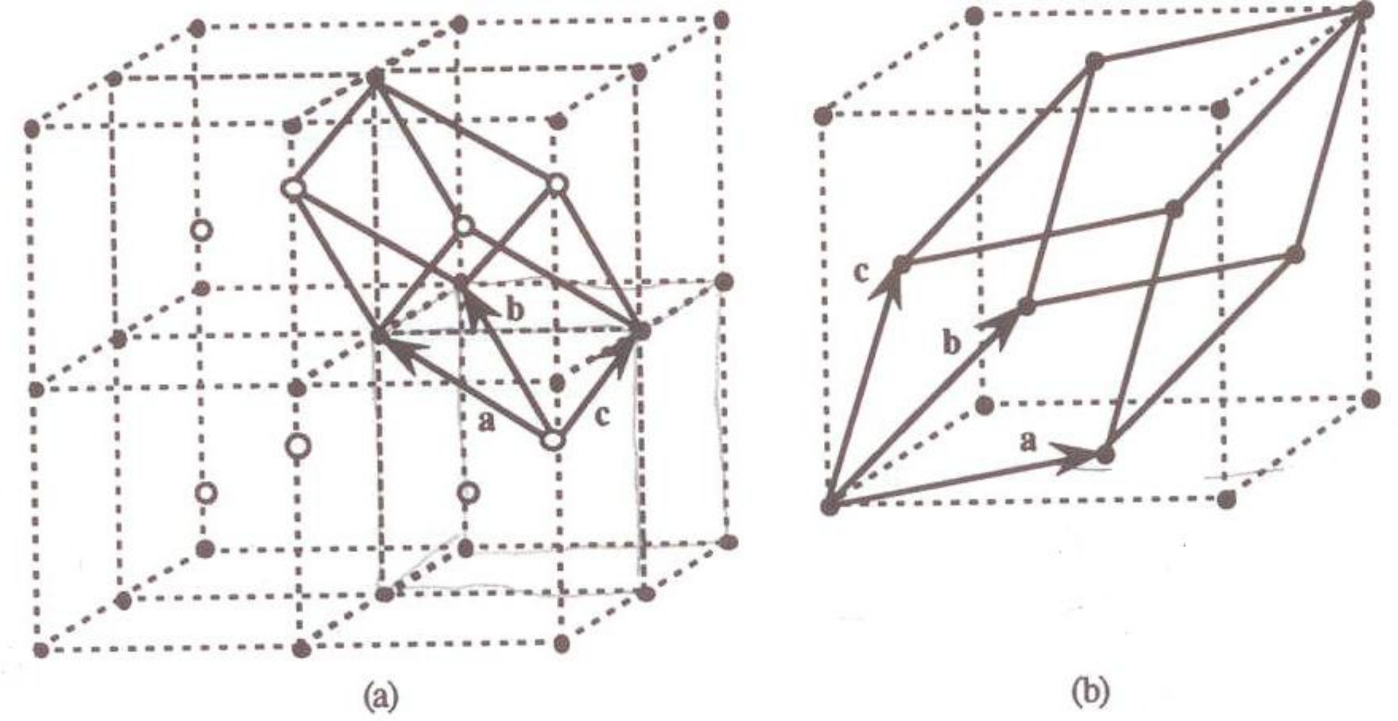


Figure 1.5 Primitive unit cells for (a) the body-centered and (b) the face-centered cubic structures. In (a) the atoms are all the same despite the fact that those at the centers of the cells are highlighted to clarify spatial relationships.

(Rhombohedron primitive unit cells)

Primitive vectors: $(a/2)(\hat{x} + \hat{y} - \hat{z})$
$(a/2)(-\hat{x} + \hat{y} + \hat{z})$
$(a/2)(\hat{x} - \hat{y} - \hat{z})$

Primitive vectors: $(a/2)(\hat{x} + \hat{y})$
$(a/2)(\hat{y} + \hat{z})$
$(a/2)(\hat{x} + \hat{z})$

Primitive unit cell of a diamond

- The space lattice of diamond cubic structure is

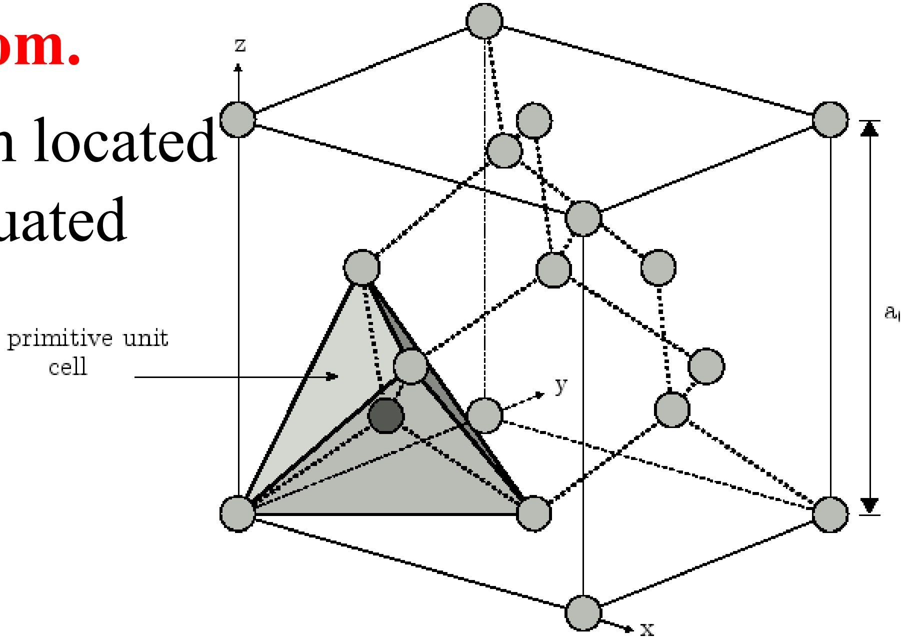
  => two inter-penetrating FCC, one displaced from the other **by a translation of a/4 (111) along a body diagonal.**

- The primitive basis has two identical atoms at 000, ¼ ¼ ¼ .

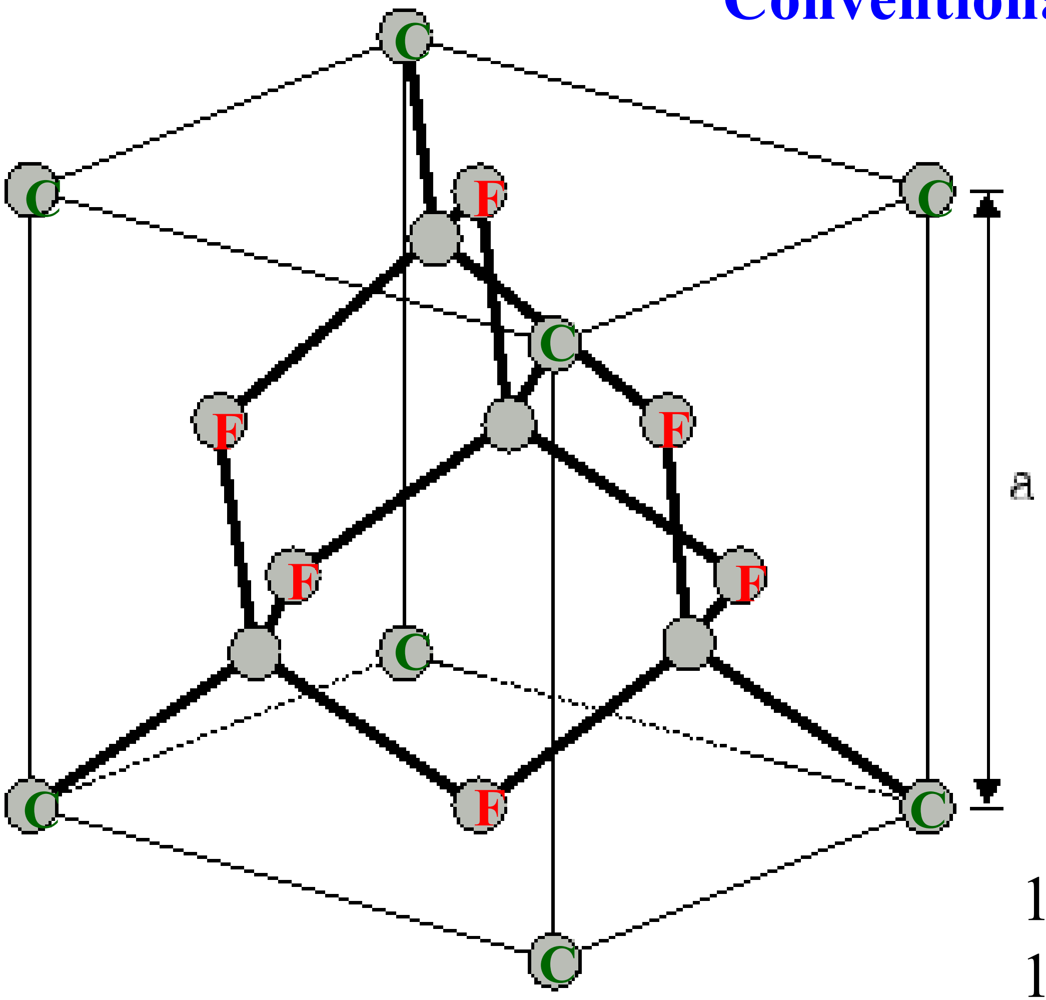
- The **conventional unit cube contains 8 atoms** (next slide).

- There is **no way** to choose the primitive cell such that **the basis of diamond contains only one atom.**

  ⇒ **a tetrahedron** with one atom located in the center and four atoms situated at the four corners.



Conventional unit cube of a diamond



Lattice = FCC (Slide 13th)

Basis = C (0,0,0), C (1/4,1/4,1/4)

1/8 × 8 (C) = 1

1/2 × 6 (F) = 3

4 atoms inside the structure

total number of atoms present in a diamond cubic unit cell is 1 + 3 + 4 = 8.

How to count lattice points?

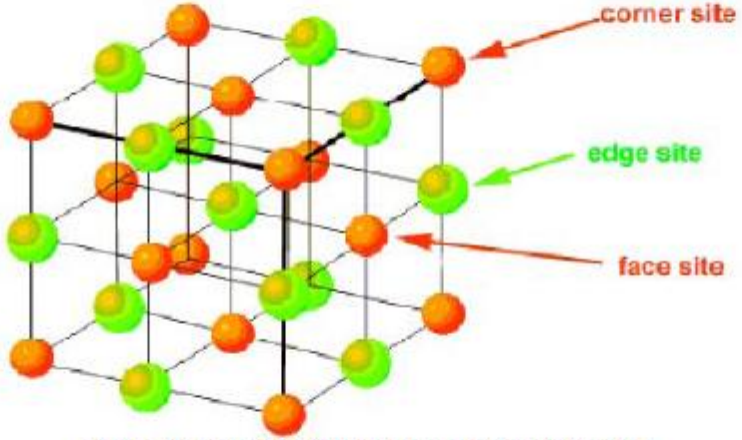
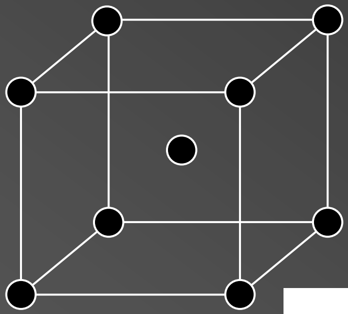


Figure 5: Different Positions in a Cubic Unit Cell

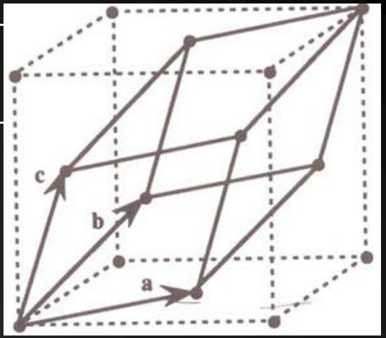
Adapted from Shriver and Atkins, "Inorganic Chemistry", 3rd edition, © W. H. Freeman.

- Corner site: eight unit cells shared (1/8).
- Edge site: four unit cells shared (1/4).
- Face site: two unit cells shared (1/2).

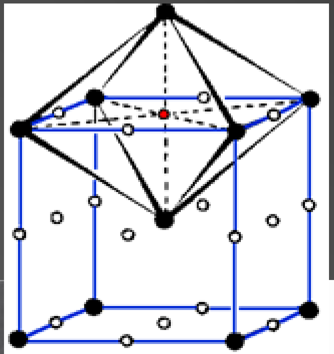
1.5 & 1.6 Body-Centered Cubic (BCC)

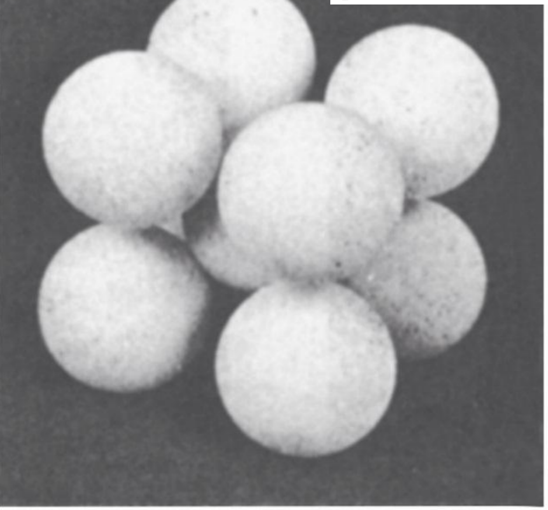


1 + (1/8) x 8 = 2 lattice points



Coordination number: the number of nearest neighbors that an atom possesses in the lattice => 8

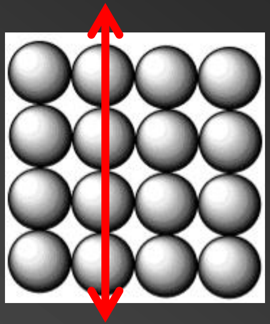


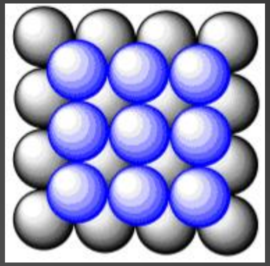


No close packed plane

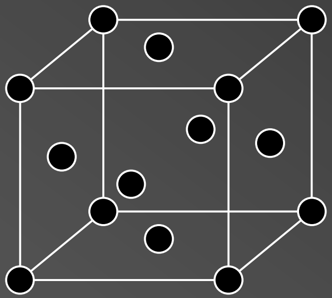
Four close packed direction <111>

FIG. 1.3 Hard-ball model of the body-centered cubic unit cell

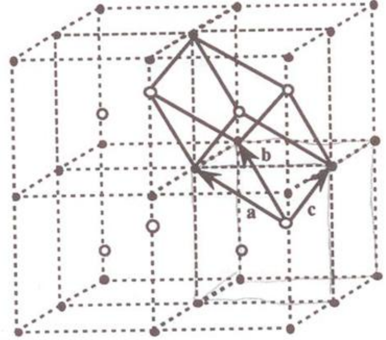


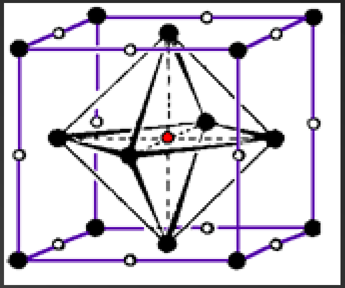


1.7 Face-Centered Cubic (FCC)



(1/8) x 8 + (1/2) x 6 = 4 lattice points





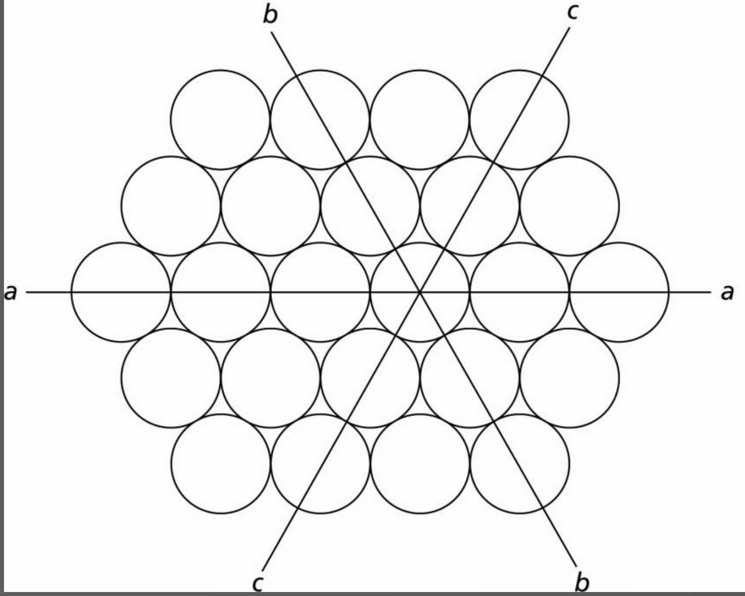


FIG. 1.5 Atomic arrangement in the octahedral plane of a face-centered cubic metal. Notice that the atoms have the closest possible packing. This same configuration of atoms is also observed in the basal plane of close-packed hexagonal crystals. The close-packed directions are *aa*, *bb*, and *cc*

octahedral planes.
(close packed plane)

Close packed direction

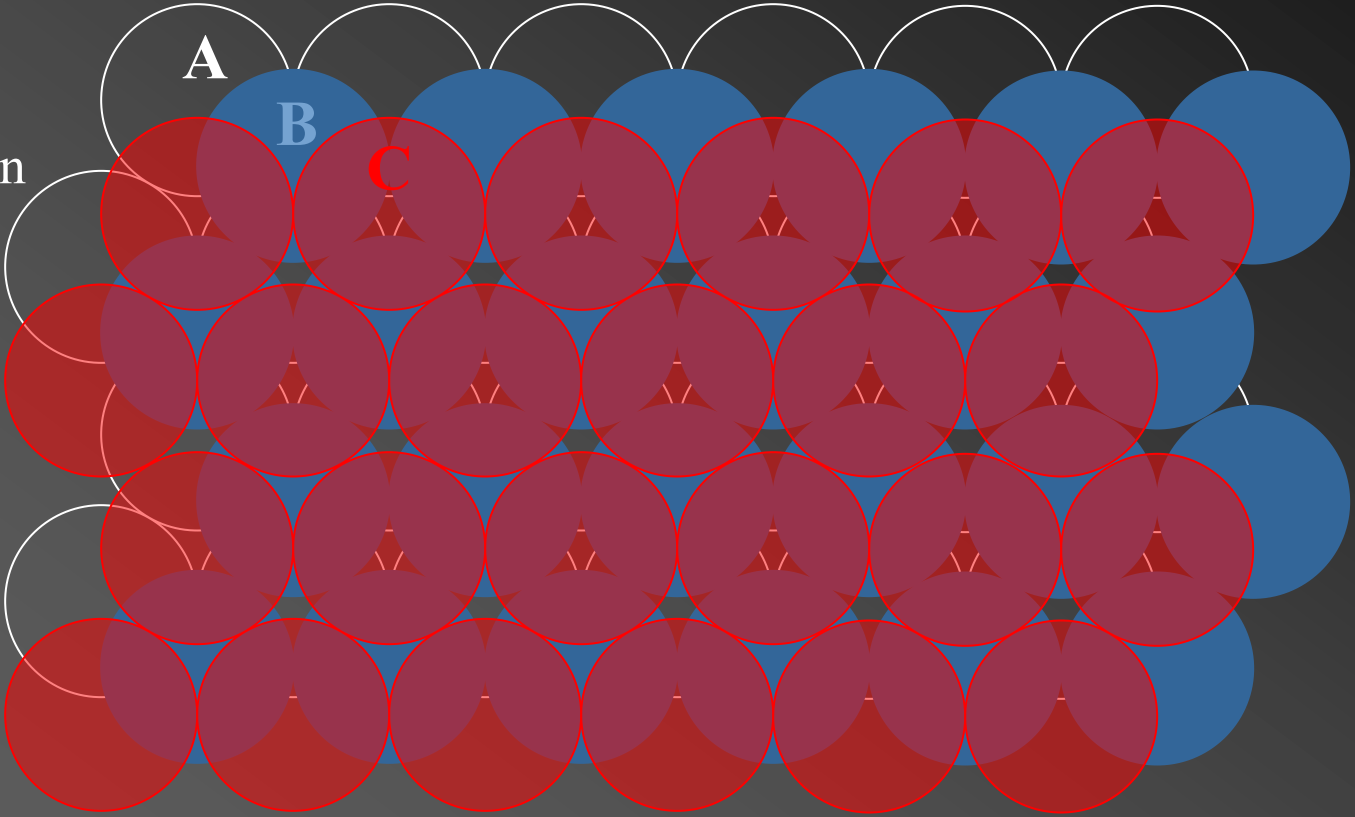
C	3
B	6
A	3

Face-Centered Cubic (FCC)

Coordination number = 12

View direction

⦿[111]



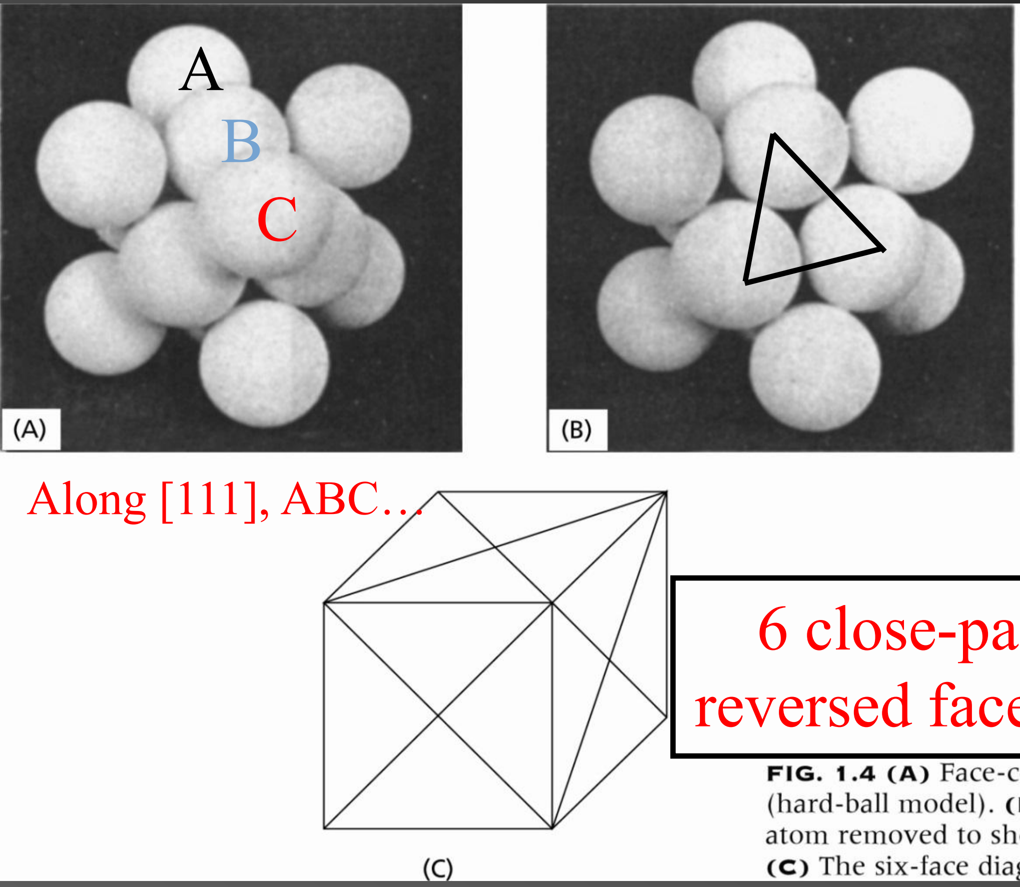
(54.75°)

A	3
B	6
A	3

Close-packed hexagonal (HCP)

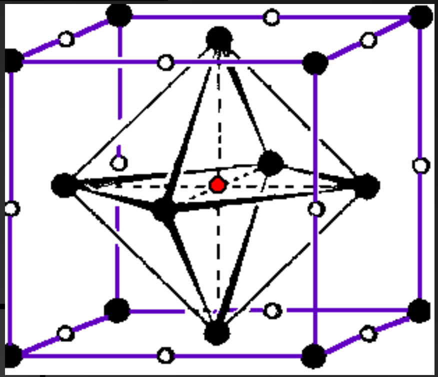
Coordination number = 12

[001] FCC



Along [111], ABC…

Close-packed plane
(octrahedral plane)
4 octrahedral planes



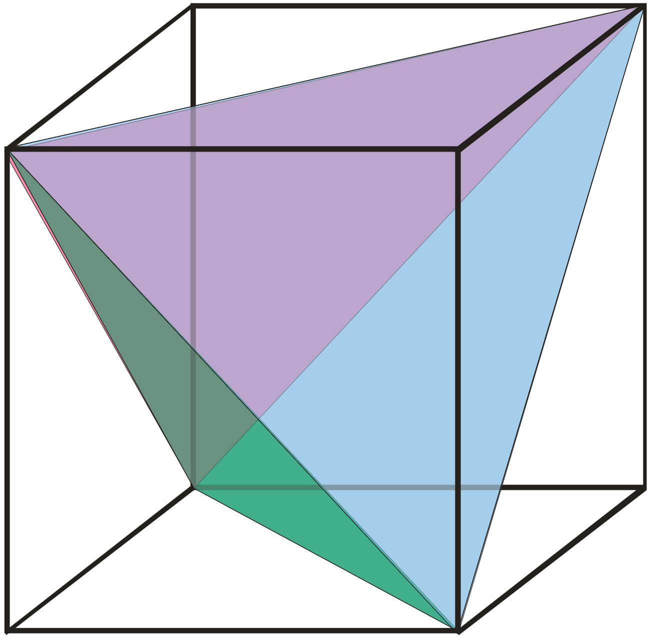
6 close-packed directions
reversed faces are not counted

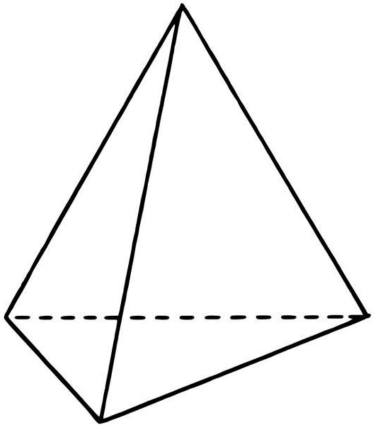
FIG. 1.4 (A) Face-centered cubic unit cell (hard-ball model). **(B)** Same cell with a corner atom removed to show an octahedral plane. **(C)** The six-face diagonal directions

No other lattice possesses such a large number of close-packed planes and close-packed directions.

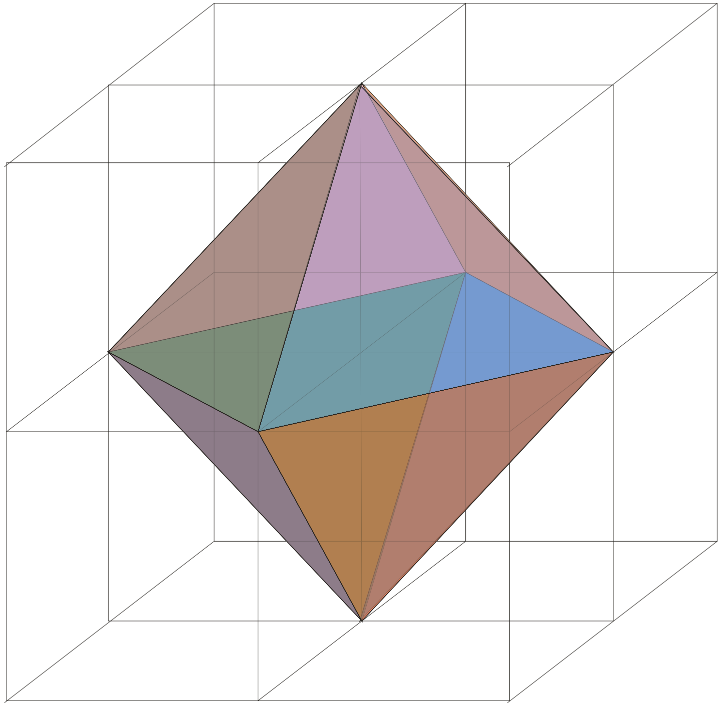
This gives FCC metals physical properties different from those of other metals, such as the ability to undergo severe plastic deformation.

Tetrahedron

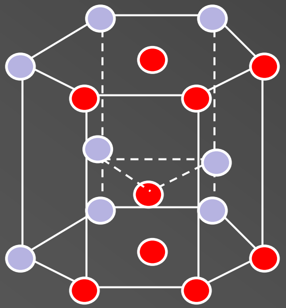




Octhedron



1.8 & 1.10 Closed-packed hexagonal (HCP) **(not a Bravais lattice)**



Hexagonal structure

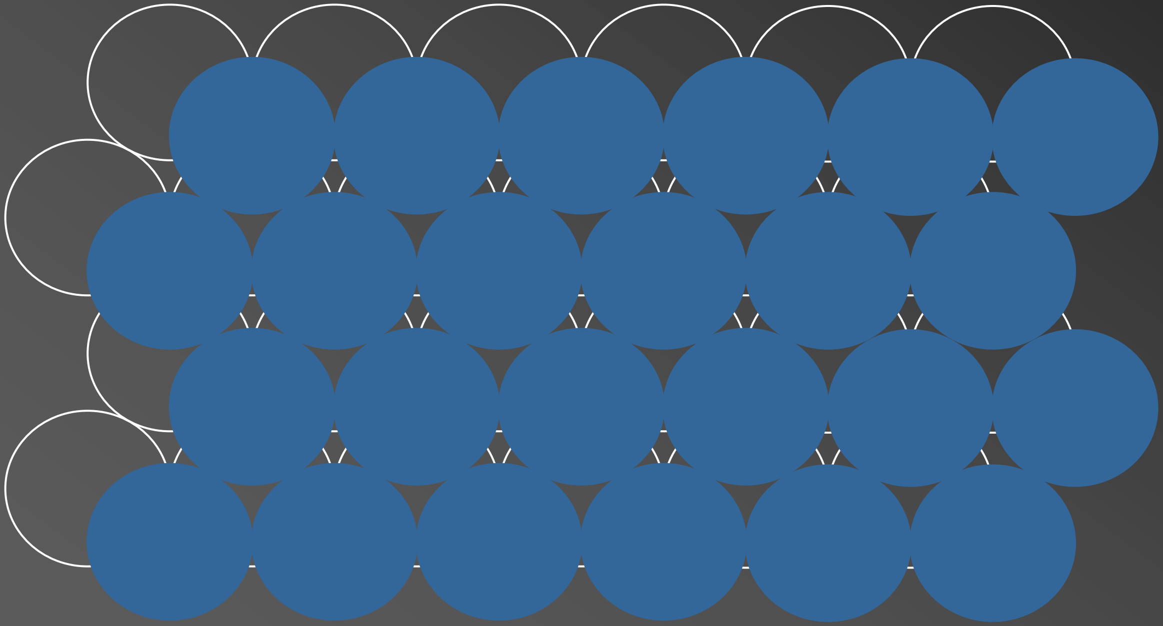
((1/6)x6x2 + (1/2)x2 + 3 = 6 lattice points)

Shows crystallographic features

Primitive unit cell: (1/8) x 8 + 1 = 2 lattice points

View direction

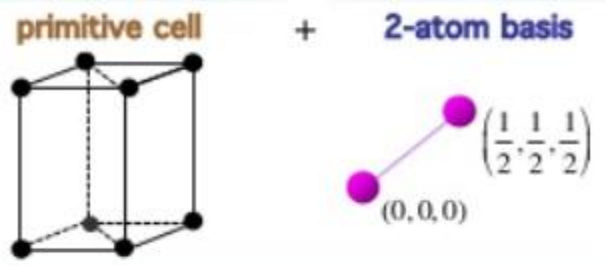
⊙[0001]

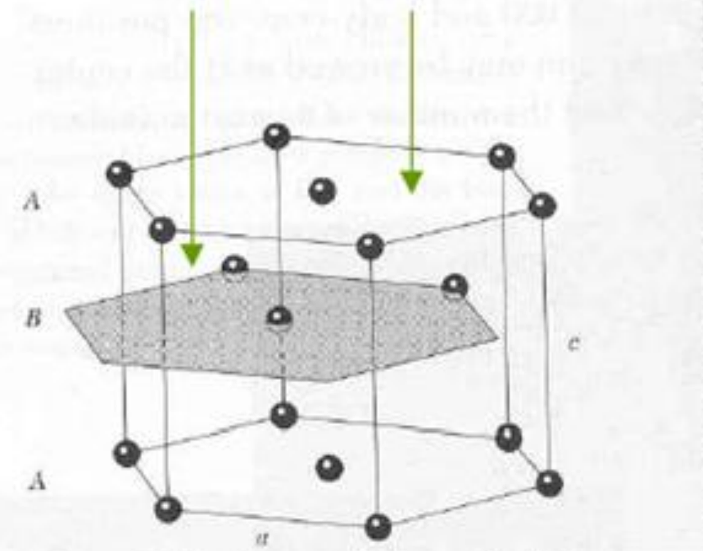


A	3
B	6
A	3

Coordination number = 12

- **HCP is not a Bravais lattice** because the orientation of the environment of a point varies from layer to layer along the c-axis.





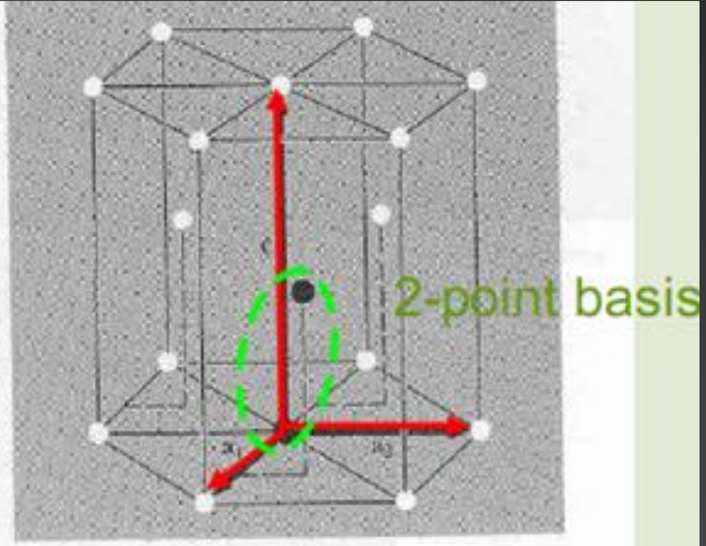


Figure 22 The hexagonal close-packed structure. The atom positions in this structure do not constitute a space lattice. The space lattice is simple hexagonal with a basis of two identical atoms associated with each lattice point. The lattice parameters *a* and *c* are indicated, where *a* is in the basal plane and *c* is the magnitude of the axis $\mathbf{a}_3$ of Fig. 14.

Figure 23 The primitive cell has $a_1 = a_2$, with an included angle of 120°. The *c* axis (or $\mathbf{a}_3$) is normal to the plane of $\mathbf{a}_1$ and $\mathbf{a}_2$. The ideal hcp structure has $c = 1.633\, a$. The two atoms of one basis are shown as solid circles. One atom of the basis is at the origin; the other atom is at $\frac{2}{3}\frac{1}{3}\frac{1}{2}$, which means at the position $\mathbf{r} = \frac{2}{3}\mathbf{a}_1 + \frac{1}{3}\mathbf{a}_2 + \frac{1}{2}\mathbf{a}_3$.

1.9 Relation between FCC (Face centered cubic) and Hexagonal (HCP) structure is the stacking sequence.

For FCC (along [111]), the stacking sequence is ABCABC…
For HCP (along [0001]), the stacking sequence is ABABAB…

There is a marked difference between the physical properties of FCC and HCP:

FCC: 4 octahedral planes, **less directional**
HCP: 1 close packed plane (basal plane); equivalent to octahedral planes of FCC: engenders plastic deformation properties that are **more directional** than found in cubic crystals.

1.11 Isotropy and Anisotropy

- Isotropy: properties are direction independent (rarely true in crystal; likely for amorphous or fine grain materials)
- Anisotropy: properties are direction dependent

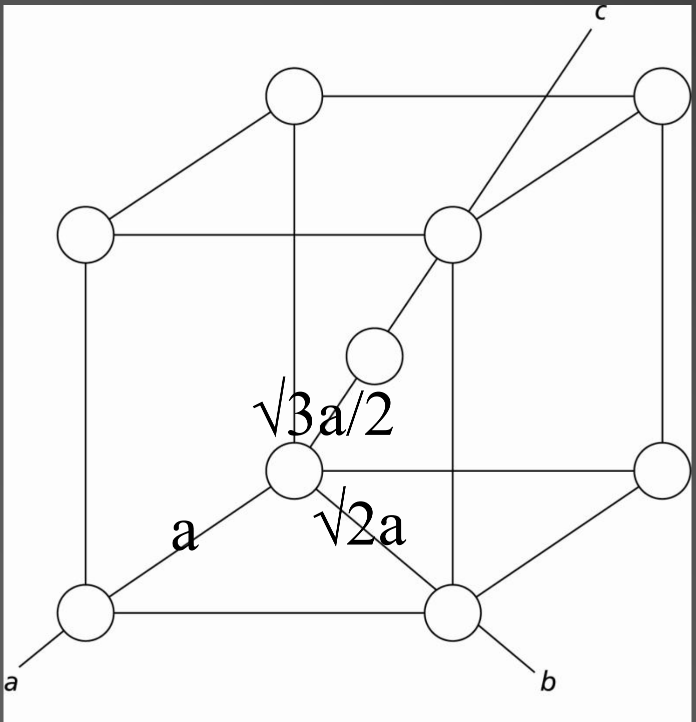


FIG. 1.8 The most important directions in a body-centered cubic crystal

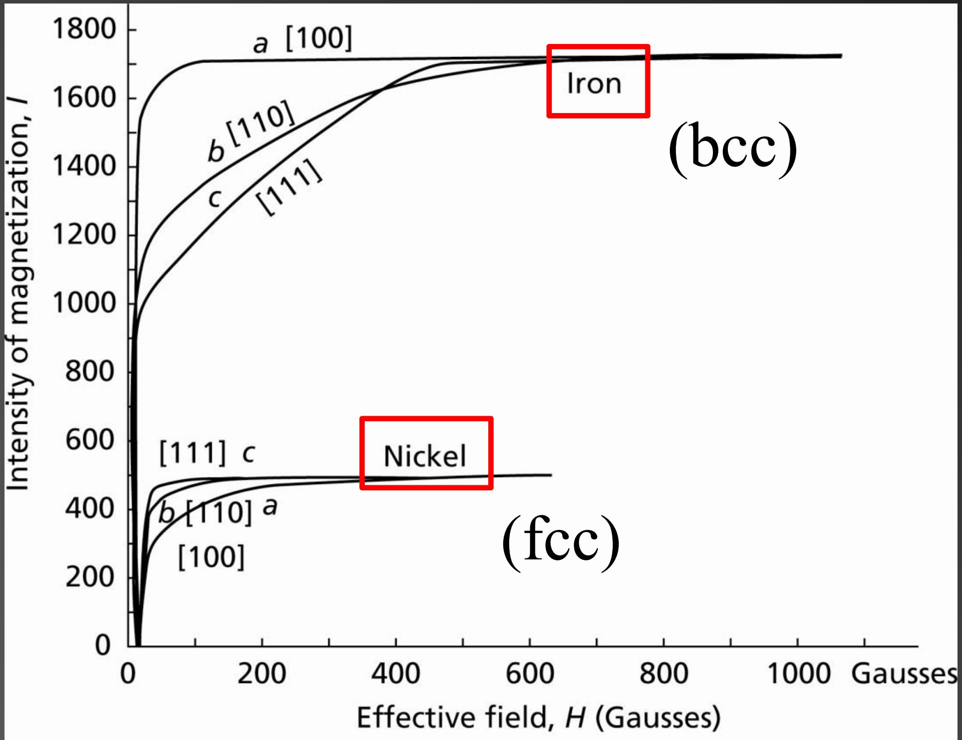


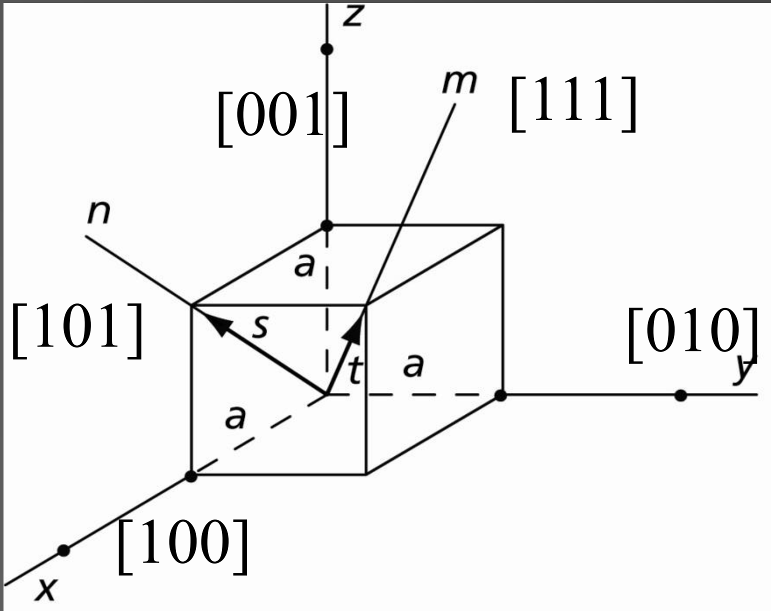
FIG. 1.9 An iron crystal is much easier to magnetize along an *a* direction of Fig. 1.8 than along a *b* or *c* direction. The opposite is the case for nickel[2,3]

1.12 Textures or Preferred Orientations

- Polycrystalline specimen might be expected isotropic if its crystals were randomly oriented.
- However, truly random arrangement of the crystals is seldom achieved.
- Processing => induced some driving forces that make crystals tends to develop in certain specific directions => **textures or preferred orientations**
- Most polycrystalline metals have a preferred orientation, they tend to be anisotropic.

1.13 Miller Indices

- For crystallographic planes and directions
- Indices for Direction: [x,y,z] represents a specific crystallographic direction; <x,y,z> represents all of the directions of the same form



Bracket [x,y,z]

Caret <x,y,z>

FIG. 1.11 The [111] and [101] directions in a cubic crystal; directions *m* and *n*, respectively

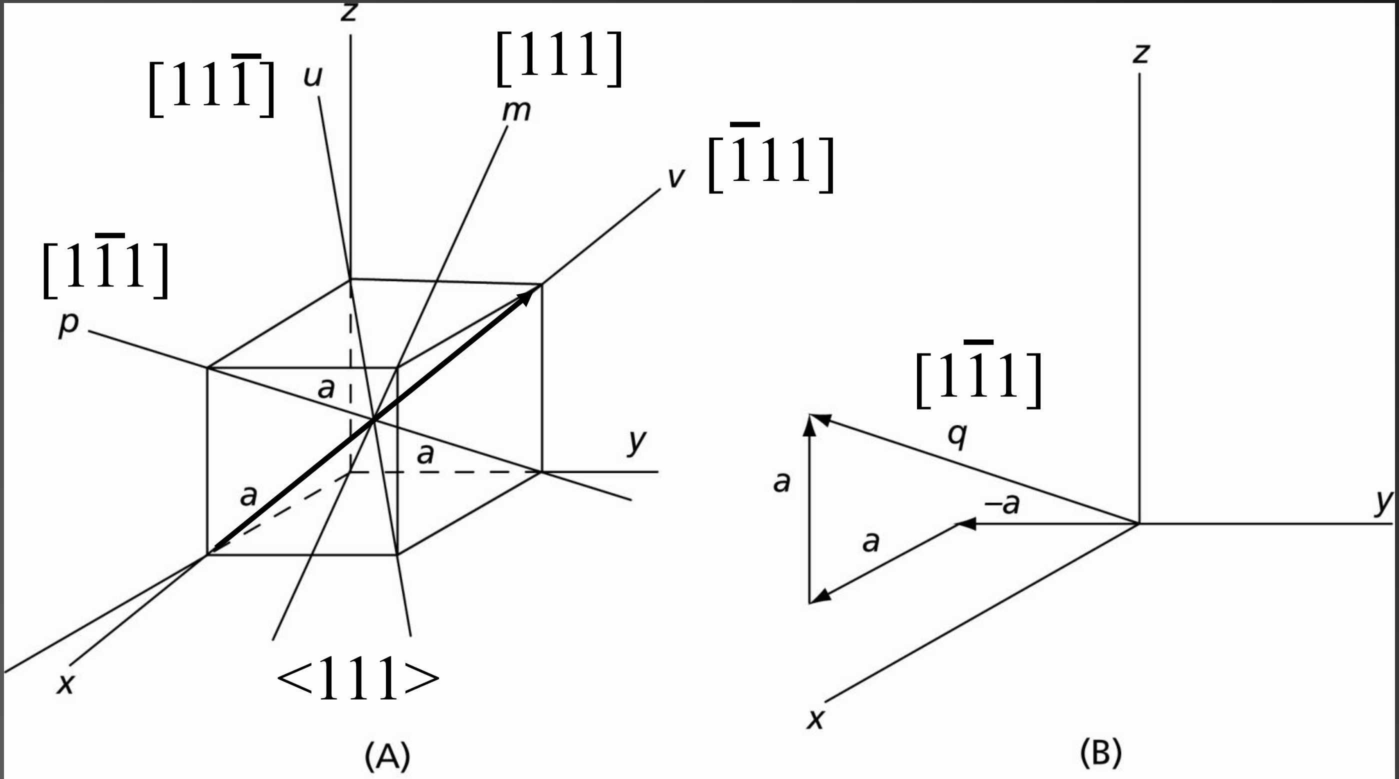


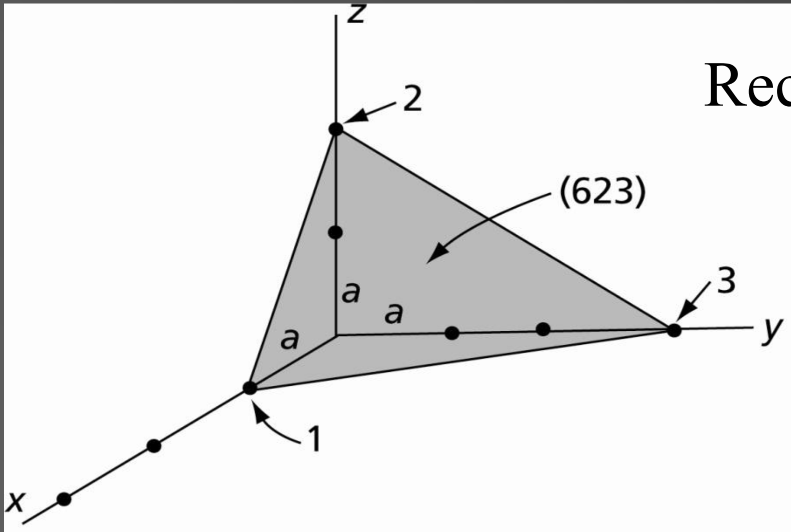
FIG. 1.12 (A) The four cube diagonals of a cubic lattice, *m*, *n*, *u*, and *v*. **(B)** The components of the vector *q* that parallels the cube diagonal *p* are *a*, −*a*, and *a*. Therefore, the indices of *q* are [111]

Four close-packed direction for BCC

- Indices for Planes: (x,y,z) represents a specific plane; {x,y,z} represents all of the planes of the same form. [x,y,z] is the directional normal of plane (x,y,z)

Parentheses (x,y,z)

Braces {x,y,z}



Reciprocal of intercepts (1/1 1/3 1/2)

(623): the smallest integers

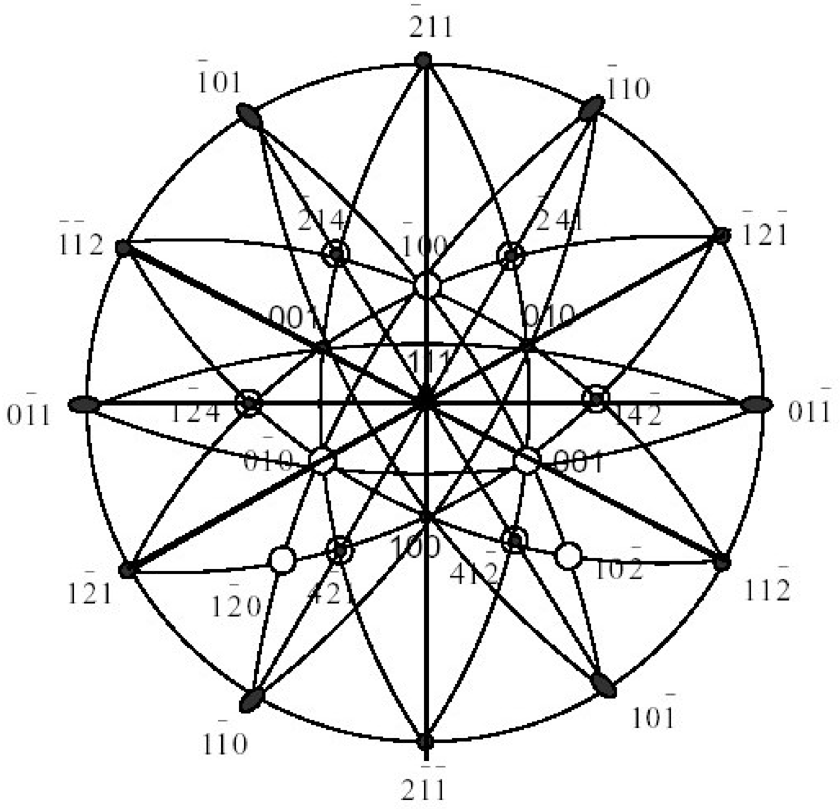
[623]: normal of (623)

Noncubic crystals do not!

FIG. 1.13 The intercepts of the (623) plane with the coordinate axes

Noncubic crystals (hkl) $\not\perp$ [hkl]

Trigonal



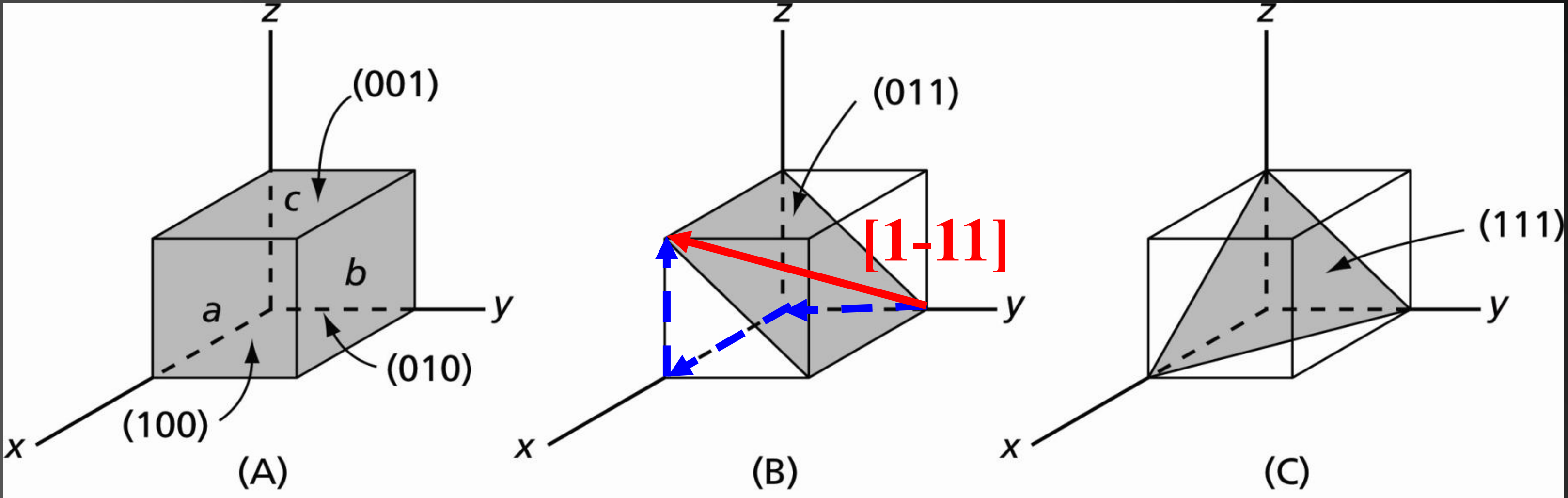


FIG. 1.14 (A) Cube planes of a cubic crystal: *a* (100); *b* (010); *c* (001). **(B)** The (011) plane. **(C)** The (111) plane

$(1/1\ 1/\infty\ 1/\infty)$	$(1/\infty\ 1/1\ 1/1)$	$(1/1\ 1/1\ 1/1)$
(100)	(011)	(111)

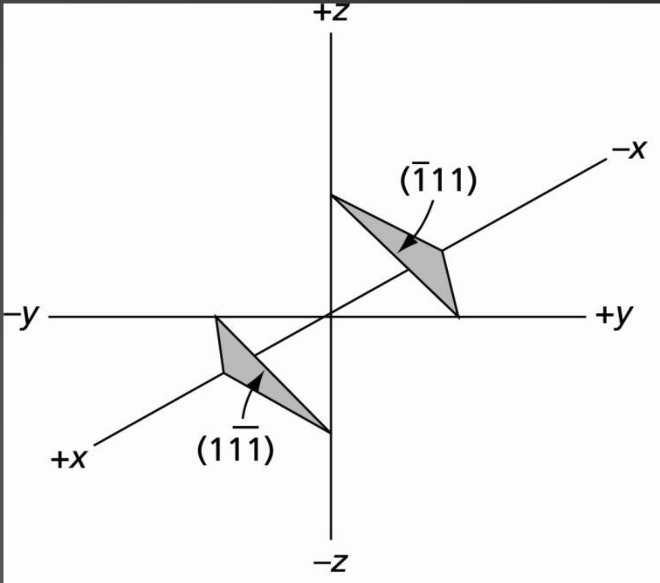


FIG. 1.15 The $(\bar{1}11)$ and $(1\bar{1}\bar{1})$ planes are parallel to each other and therefore represent the same crystallographic plane

$(111) \parallel (\bar{1}\bar{1}\bar{1})$

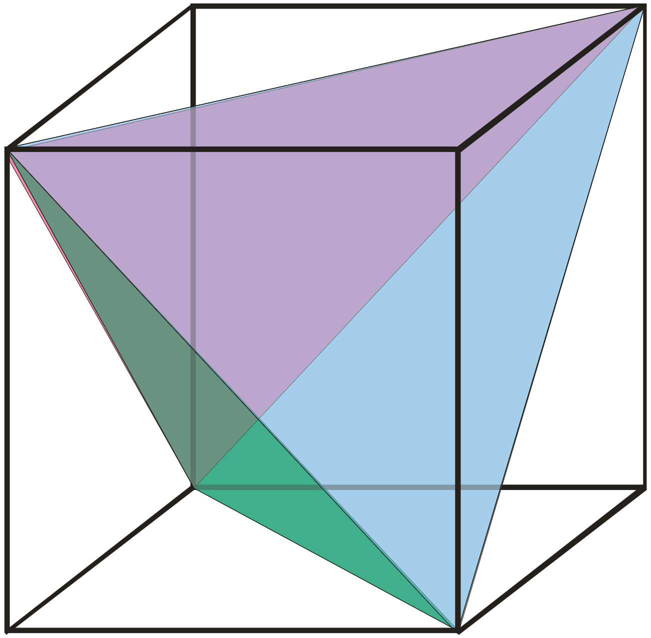
$(\bar{1}11) \parallel (1\bar{1}\bar{1})$

$(1\bar{1}1) \parallel (\bar{1}1\bar{1})$

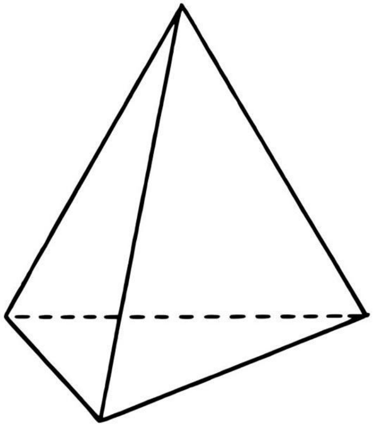
$(11\bar{1}) \parallel (\bar{1}\bar{1}1)$

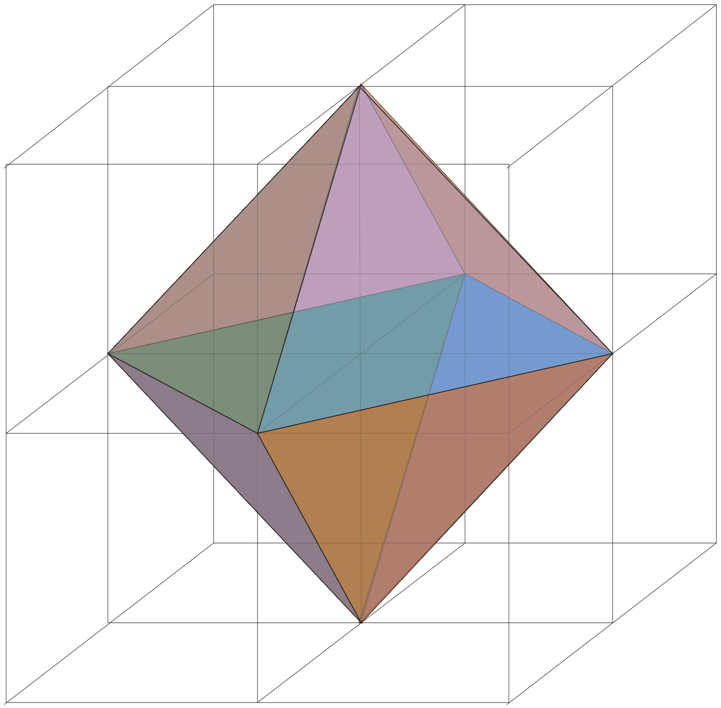
Group {111}

Index	Number of members in a cubic lattice	d_{hkl}	
(100)	6 {100}	$d_{100} = a$	
(110)	12 {110}	$d_{110} = a/\sqrt{2} = a\sqrt{2}/2$	The (110) plane bisects the face diagonal
(111)	8 {111}	$d_{111} = a/\sqrt{3} = a\sqrt{3}/3$	The (111) plane trisects the body diagonal
(210)	24 {210}		
(211)	24 {211}		
(221)	24 {221}		
(310)	24 {310}		
(311)	24 {311}		
(320)	24 {320}	(6x4x2)/2	
(321)	48 {321}	(6x4x2)	



Tetrahedron inscribed inside a cube with bounding planes belonging to the {111} family





8 planes of {111} family forming a regular octahedron

1.13 Miller-Bravais Indices for Hexagonal

- Miller-Bravais Indices for Hexagonal Crystals: 4-digit system (*hklm*)
- Families of planes are apparent, ex $(11\bar{2}0)$ and $(1\bar{2}10)$

- 3-digit: $h + k = -l$ => (*hklm*) reduced to (*hkm*) hexagonal symmetry is not apparent
- Equivalent planes are not similar in 3-digit, ex.

  $(11\bar{2}0) = (110)$

  $(1\bar{2}10) = (1\bar{2}0)$

- [*hklm*] is perpendicular to (*hklm*), similar to Miller indices in cubic systems.

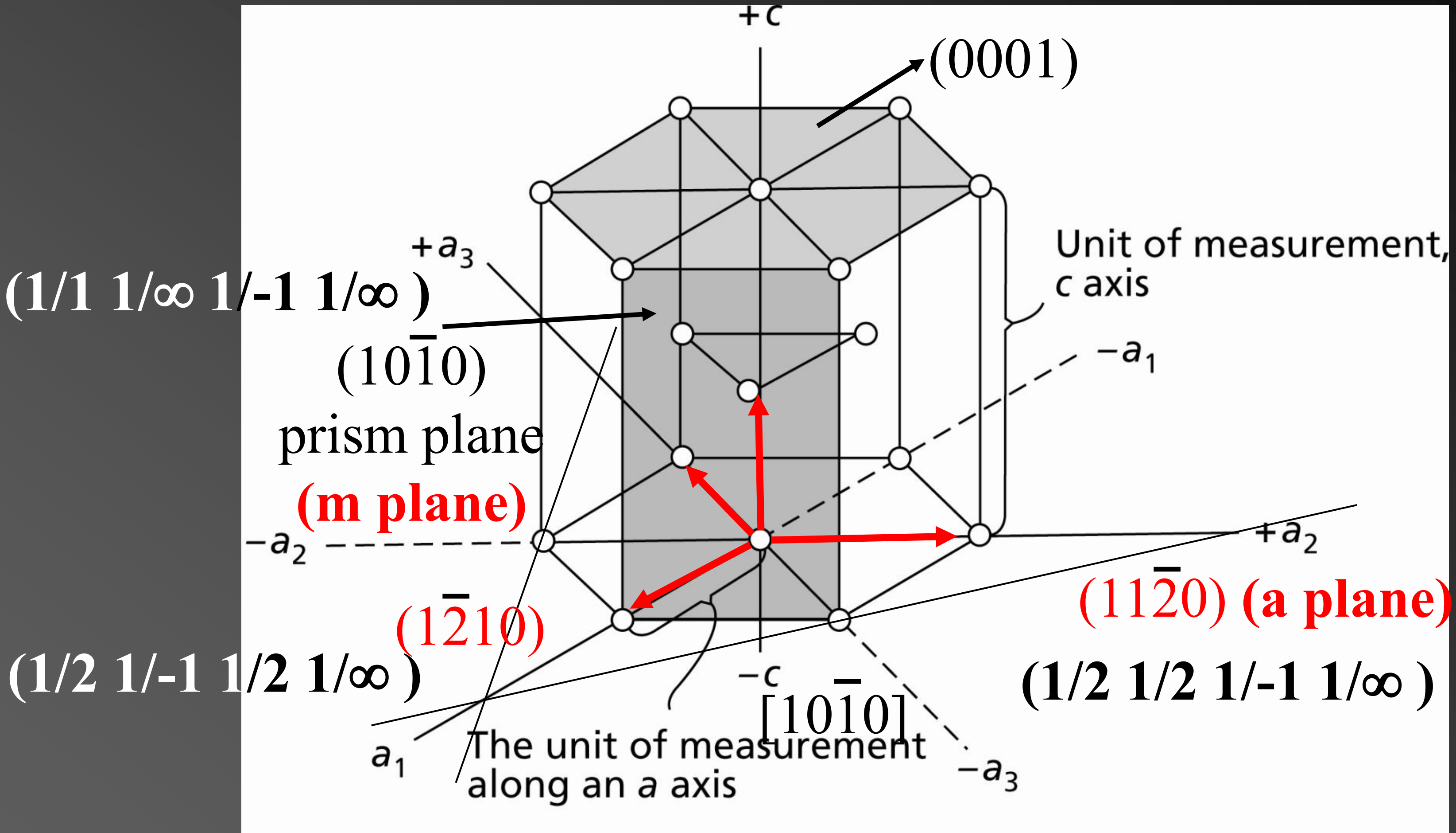


FIG. 1.16 The four coordinate axes of a hexagonal crystal

1. **Basal plane: (0001) (c plane)**
2. **6 x prism planes (type I): {1010} (m plane)**
3. **Another important plane $(10\bar{1}2)$ (next slide)**

(1/1 1/∞ 1/-1 1/(1/2))

$(10\bar{1}2)$

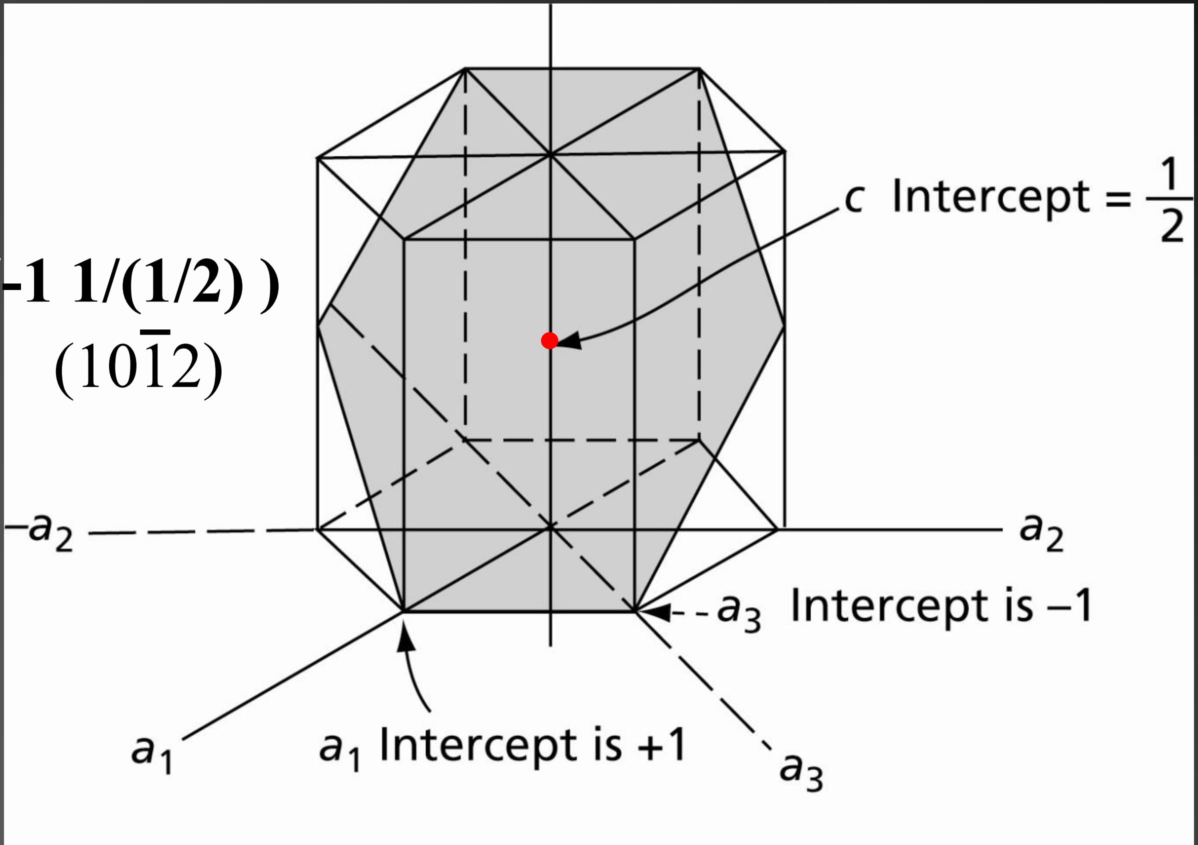


FIG. 1.17 The $(10\bar{1}2)$ plane of a hexagonal metal

(r plane)

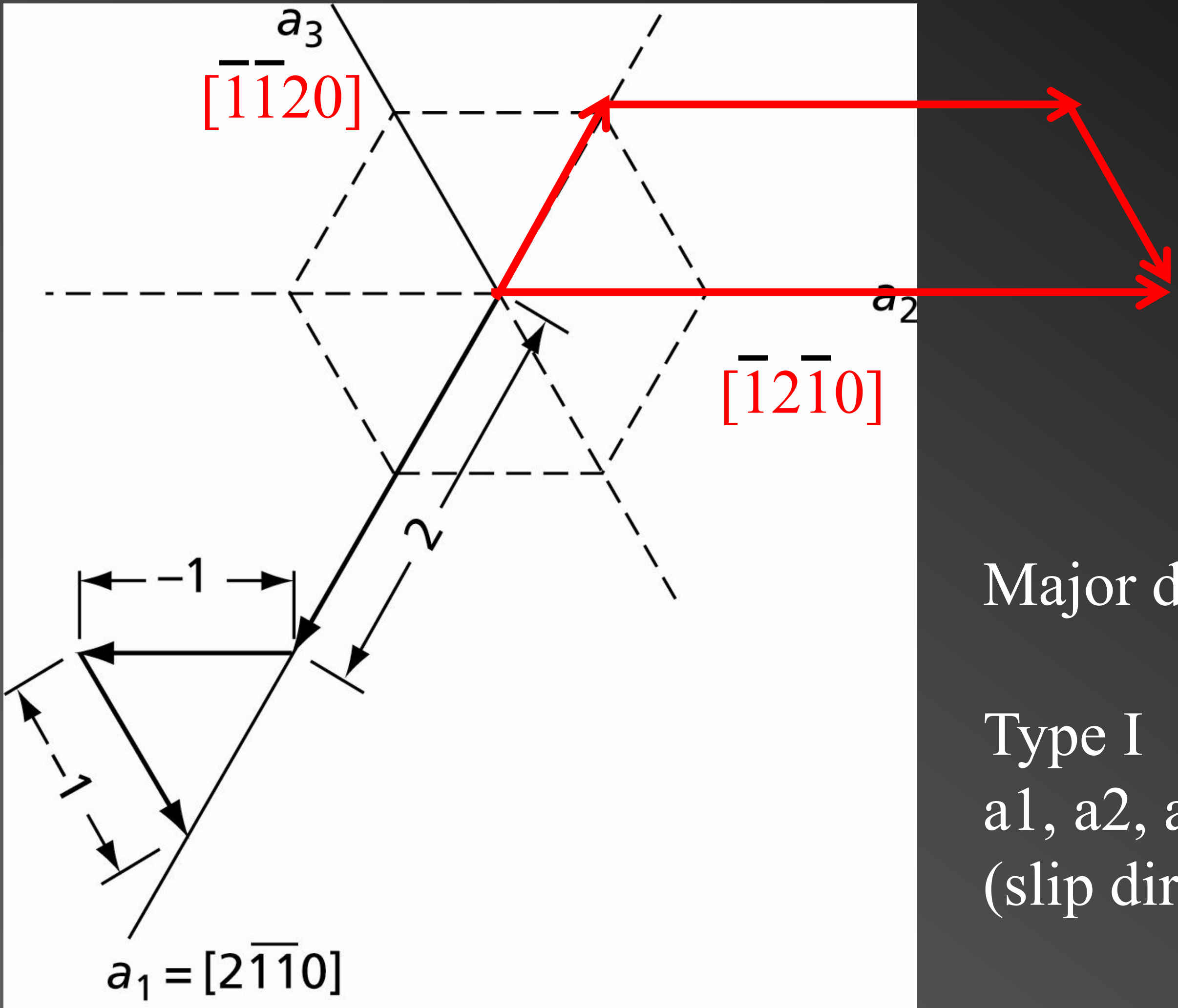


FIG. 1.18 Determination of indices of a digonal axis of Type I—$[2\bar{1}\bar{1}0]$

Major directions:

Type I
a1, a2, a3 three axes
(slip directions)

Type II (next slide)

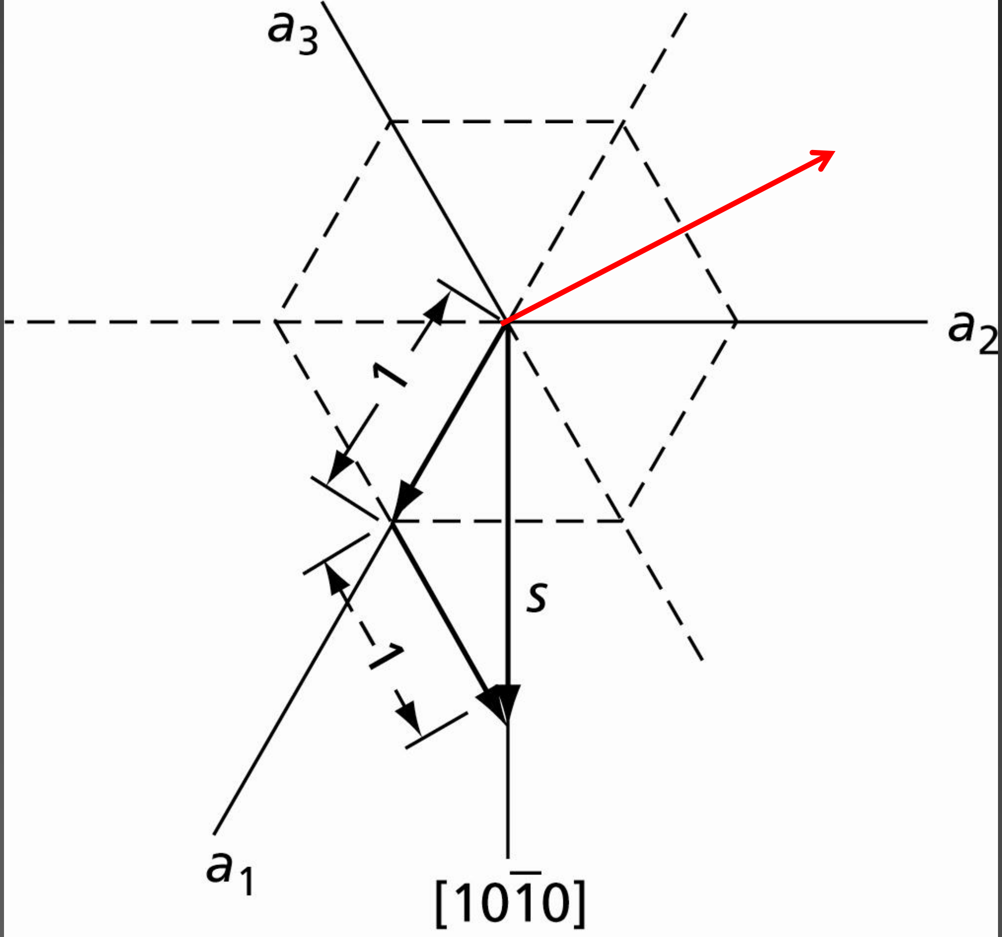


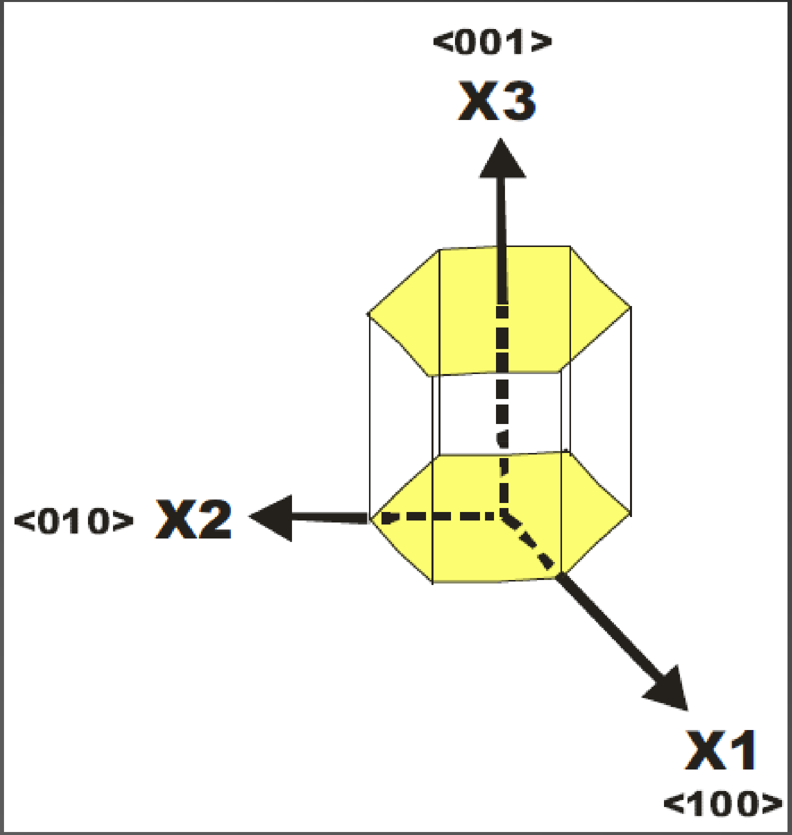
FIG. 1.19 Determination of indices of a digonal axis of Type II—[10$\bar{1}$0]

Major directions:

Type II

S [10$\bar{1}$0]

Three-index system in hexagonal



MIller-Bravais Miller

Direction <UVTW> to <uvw>:

u=U-T

v=V-T

w=W

for example: <2-1-10> is equal <100>

Miller MIller-Bravais

Direction <uvw> to <UVTW>

U=(2u-v)/3
V=(2v-u)/3
T=-(u+v)/3
W=w
for example:
<100> is equal <2-1-10>.
<210> is equal <10-10>

Ex. $(100) \Rightarrow (10\bar{1}0)$ $[10\bar{1}0]$

⇩

$(100) \perp [210]$

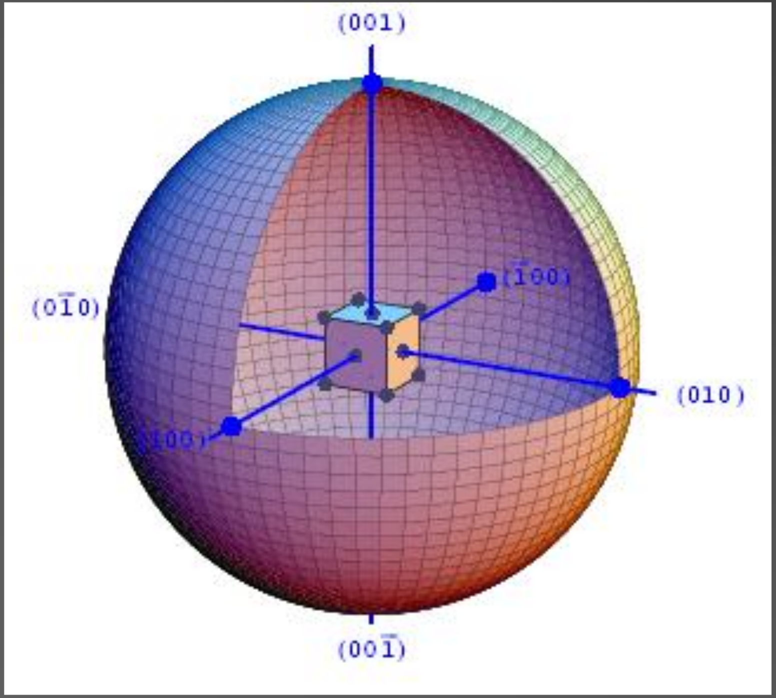
Ex. $(01\bar{1}1)$ $[01\bar{1}1]$ => (011) $[121]$

$\perp$ $\perp$

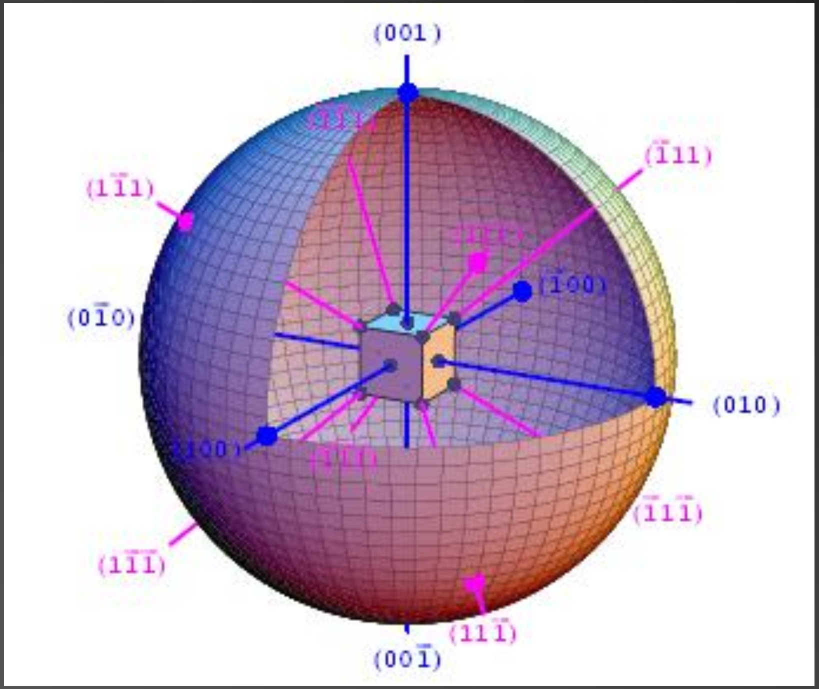
Ex. $[2\bar{1}\bar{1}0] \Rightarrow [300]$

1.15 The Stereographic Projection:

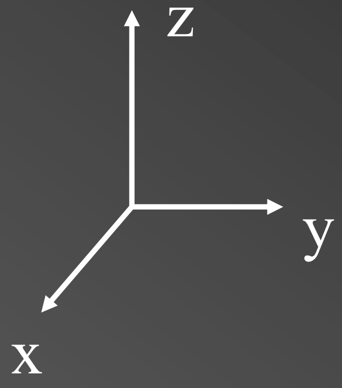
- A tool to map in two dimensions of crystallographic planes and directions.
- Planes are plotted as great **circle lines**; directions are plotted as **points**.

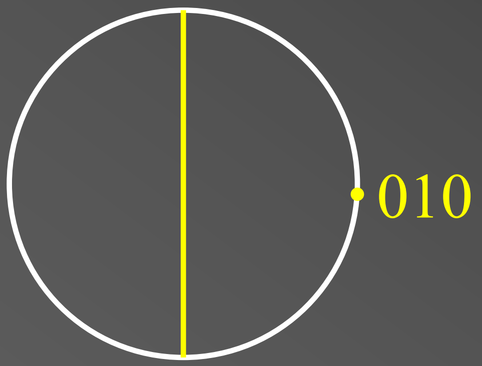


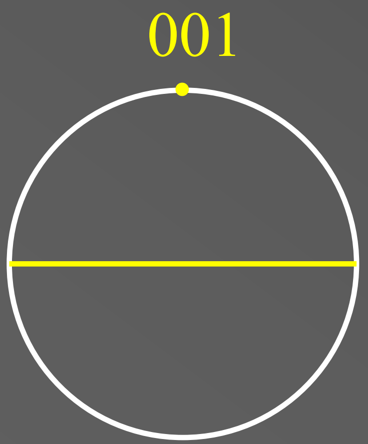
{100} Poles of a cubic crystal



+ {111} Poles







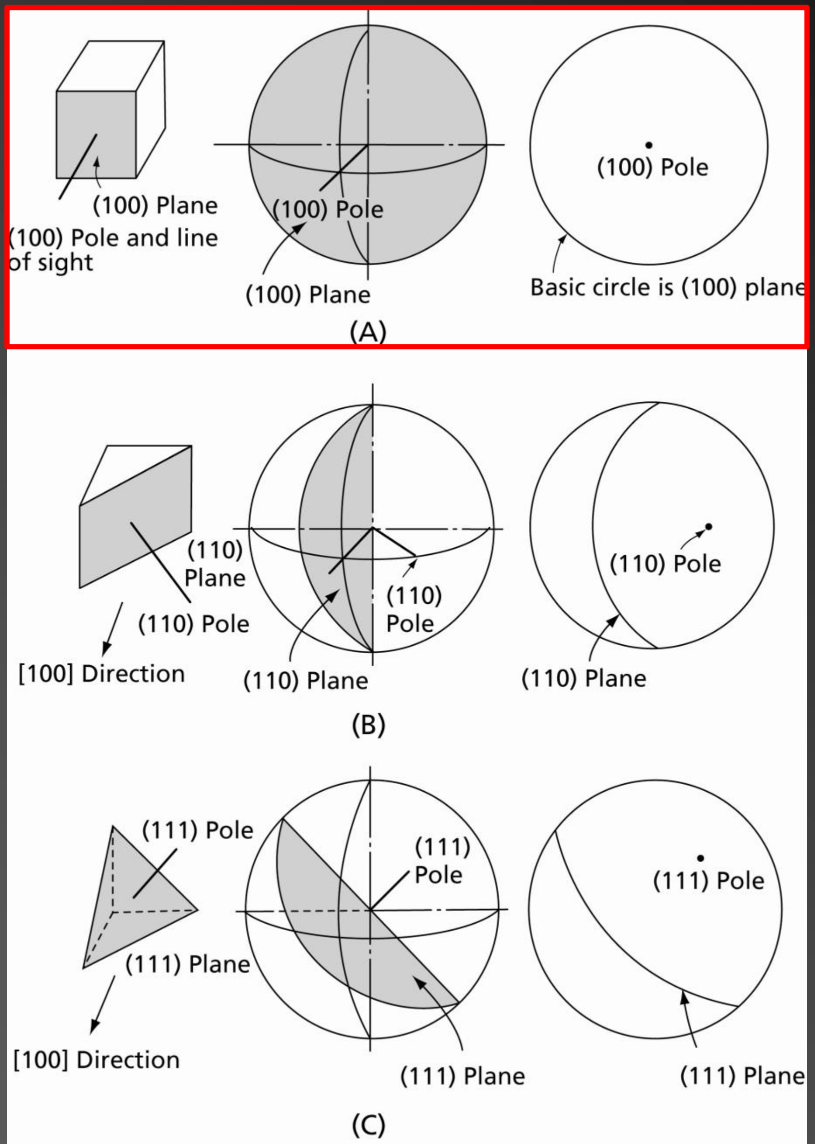
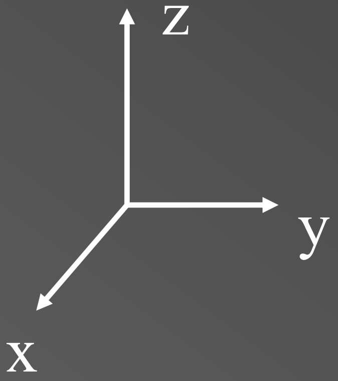


FIG. 1.20 Stereographic projections of several important planes of a cubic crystal. **(A)** The (100) plane, line of sight along the [100] direction. **(B)** The (110) plane, line of sight along the [100] direction. **(C)** The (111) plane, line of sight along the [100] direction

(100)

1.15 The Stereographic Projection:

- both hemispheres Fig. 1. 21



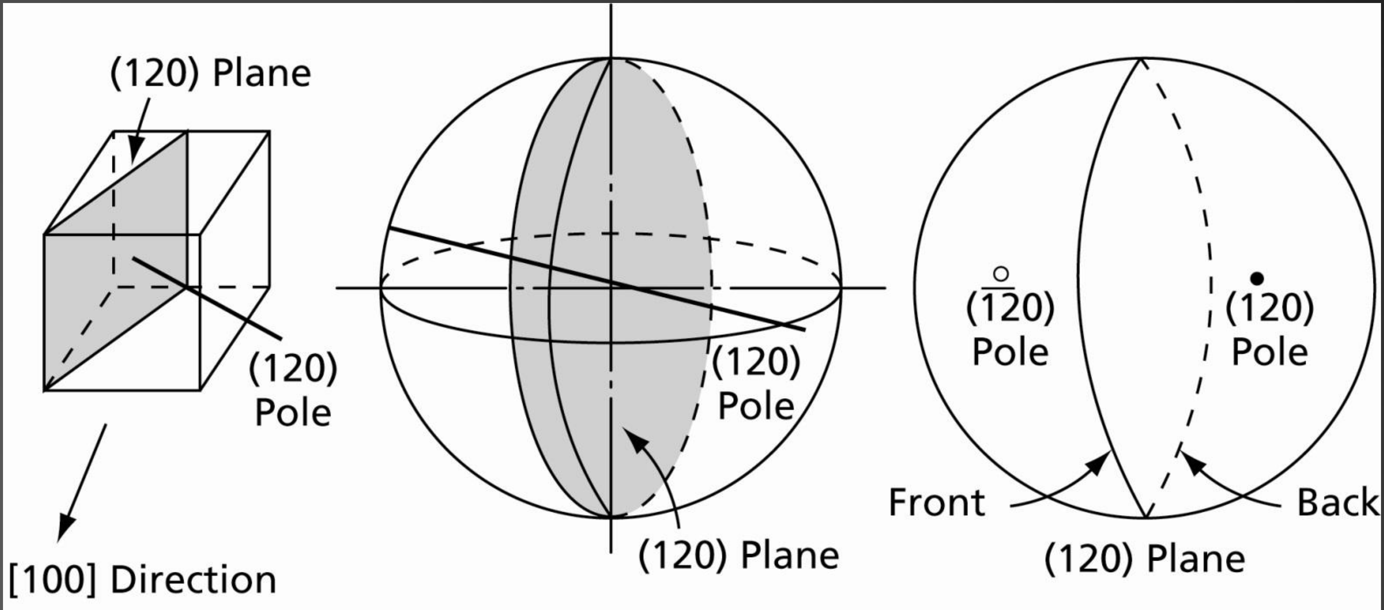
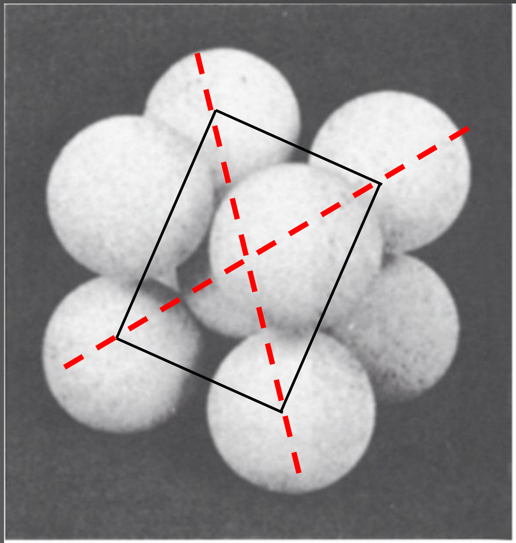
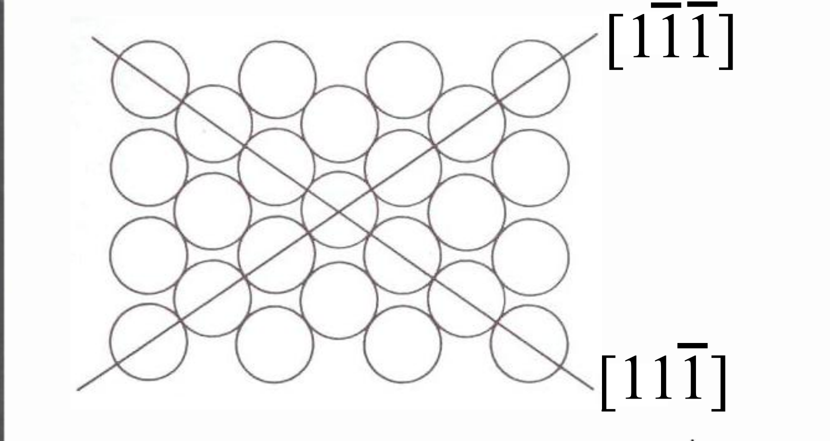


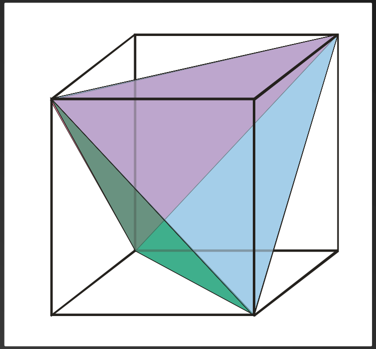
FIG. 1.21 Cubic system, the (120) plane, showing the stereographic projections from both hemispheres, line of sight the [100] direction

1.16 Directions that lie in a plane

- BCC: 4 important planes {110}, each containing two closed-packed directions <111>: total 4.







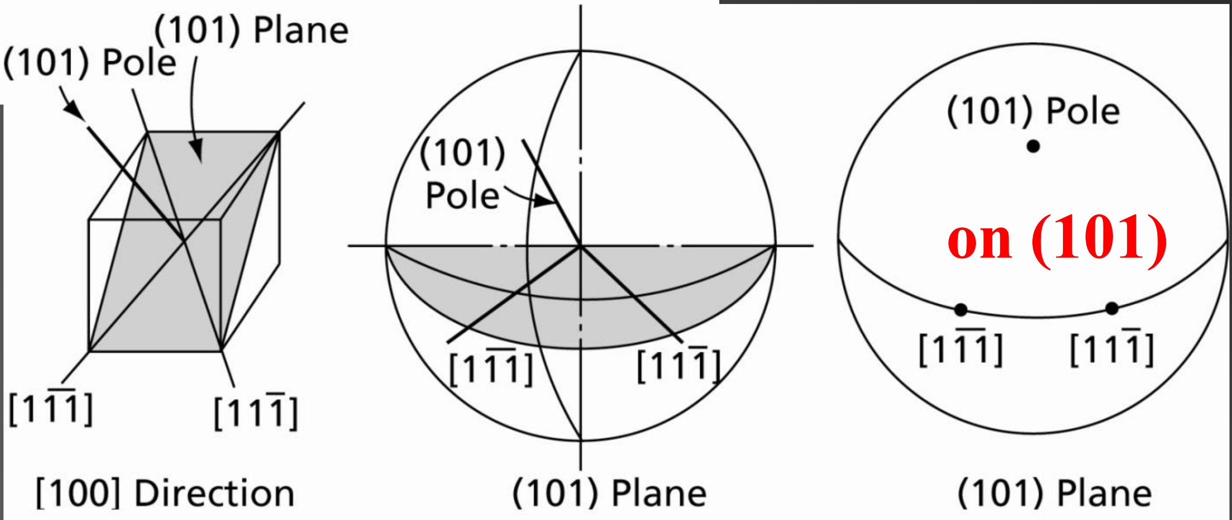
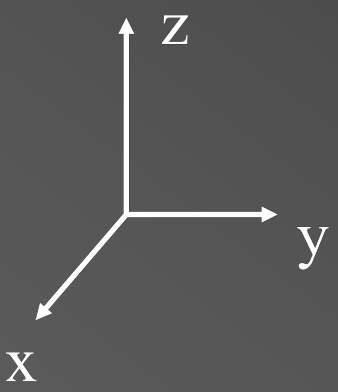
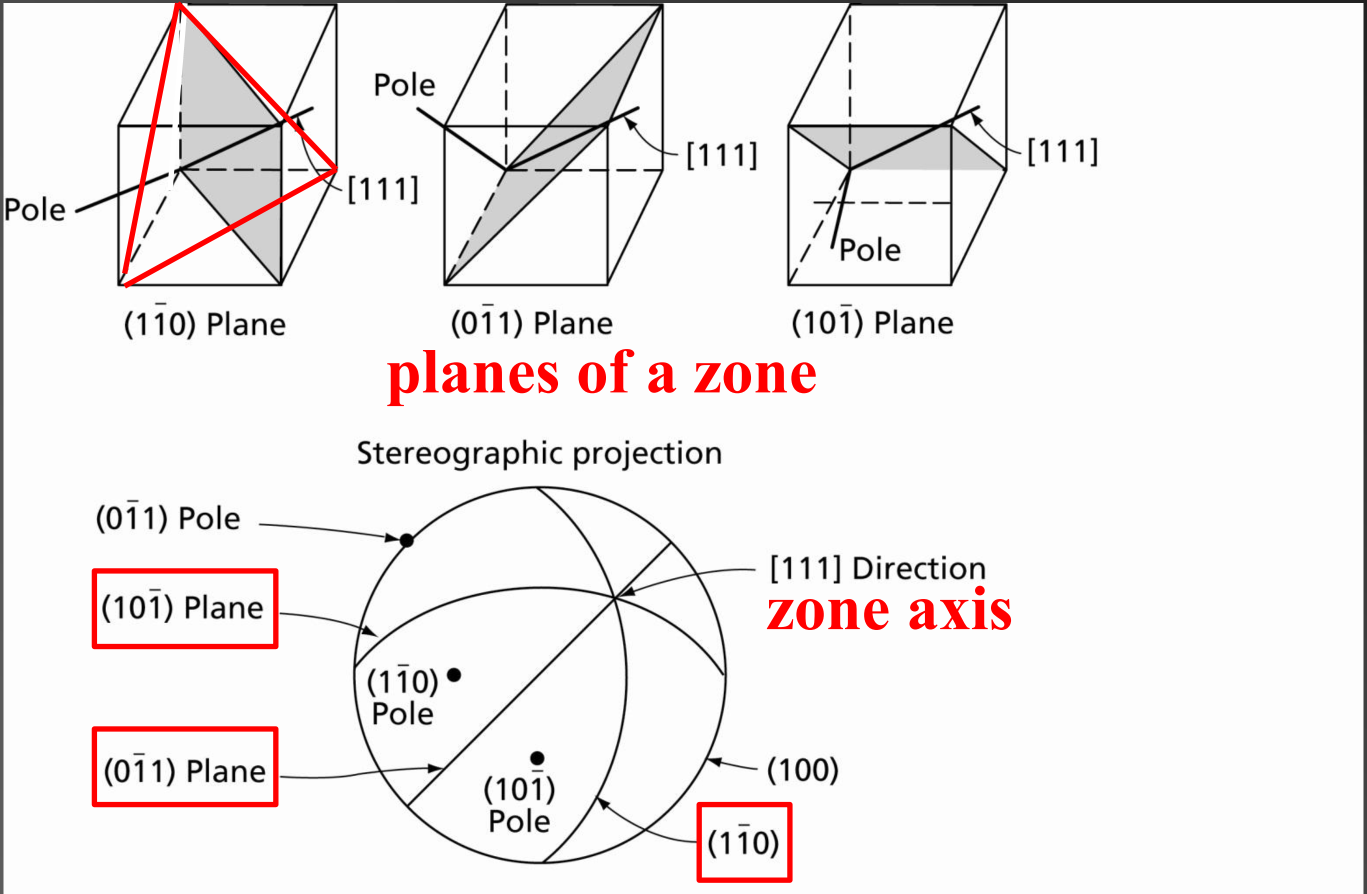


FIG. 1.22 Cubic system, the (101) plane and the two ⟨111⟩ directions that lie in this plane, line of sight [100]

1.17 Planes of a zone

- those planes that mutually intersect along a common direction form the planes of a zone
- the line of intersection is called zone axis.





planes of a zone

zone axis

FIG. 1.23 Cubic system, zone of planes the zone axis of which is the [111] direction. The three {110} planes that belong to this zone are illustrated in the figures

1.17 Planes of a zone

- 3 {112} and 6 {123} planes have the same zone axis [111].

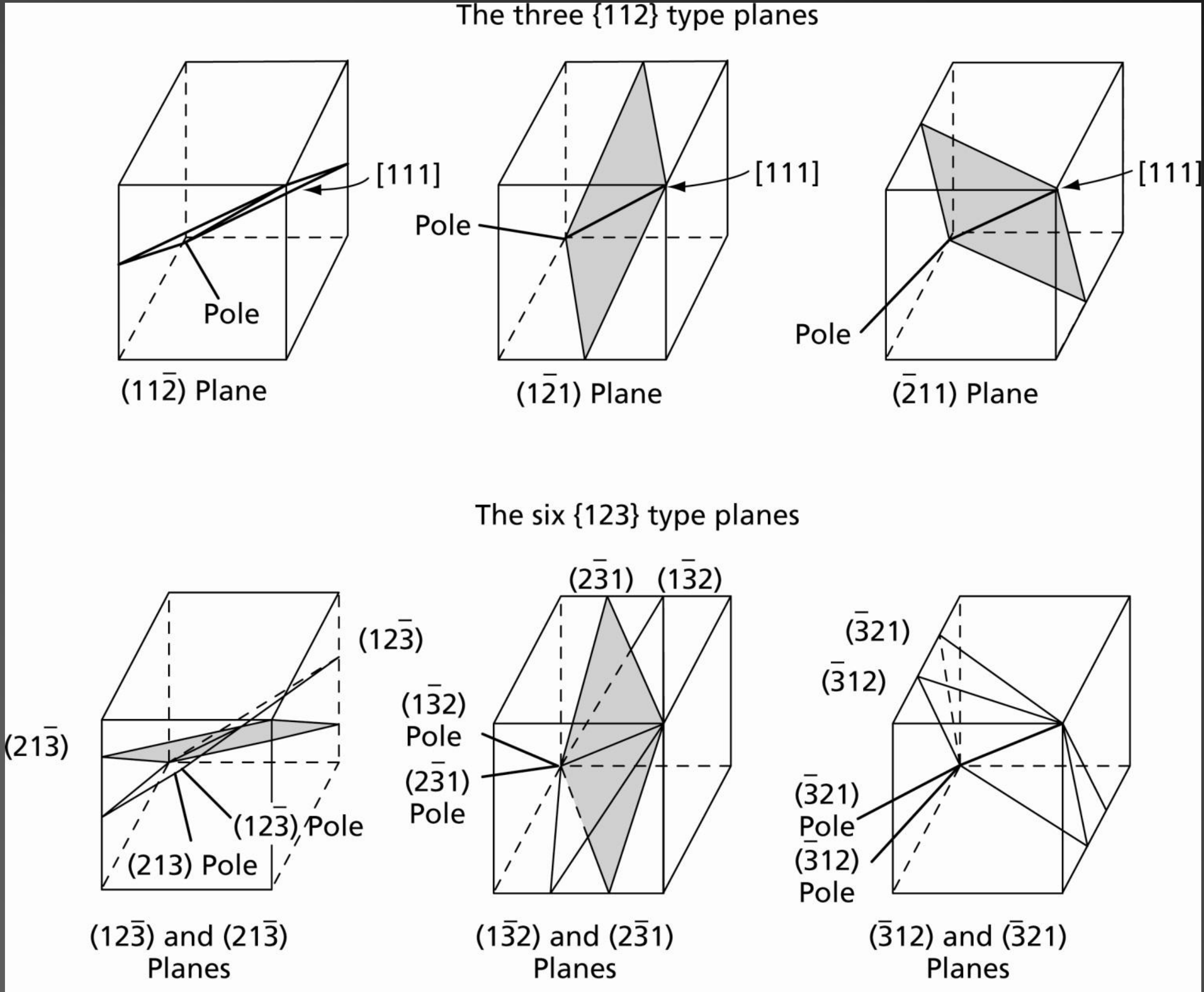
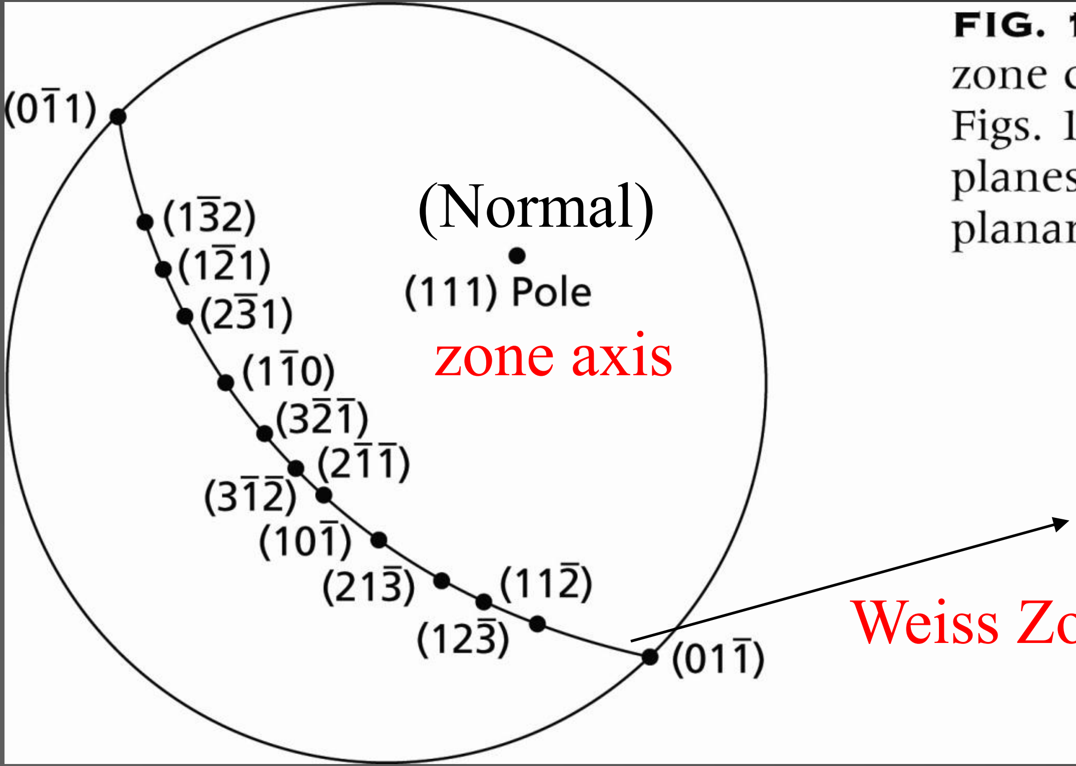


FIG. 1.24 The {112} and {123} planes that have [111] as their zone axis

1.17 Planes of a zone

- Stereographic projection: only the poles of the planes are plotted
- All of 13 planar poles lie in the (111) plane



(Normal)

zone axis

FIG. 1.25 Stereographic projection of the zone containing the 12 planes shown in Figs. 1.23 and 1.24. Only the poles of the planes are plotted. Notice that all of the planar poles lie in the (111) plane

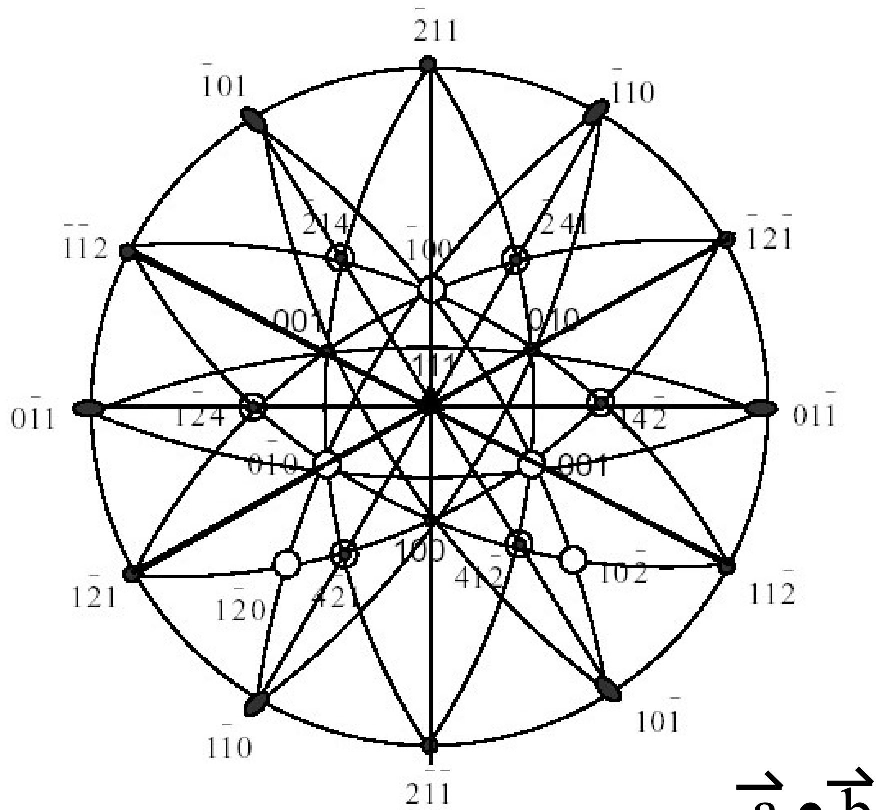
(111)

Weiss Zone Law (for any crystals)

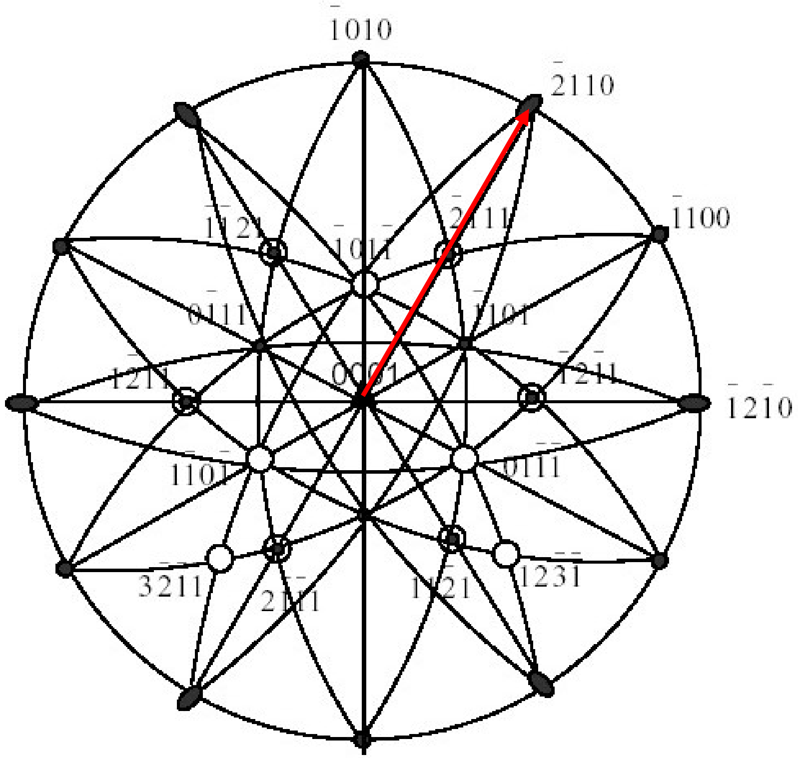
$$\vec{a} \bullet \vec{b} = 0$$

Noncubic crystals (hkl) $\cancel{\perp}$ [hkl]

Trigonal



Hexagonal



$$\vec{a} \bullet \vec{b} = 0$$

Planes of a zone and zone axis

1.18 The Wulff Net

- The Wulff net is a stereographic projection of latitude and longitude lines in which the north-south axis is parallel to the plane of the paper.
- We are primarily interested in measuring angles.
- In the handling of many crystallographic problems, it is frequently necessary to rotate a stereographic projection corresponding to a given crystal orientation into a different orientation.

  ⇒ To bring experimentally measured data into a standard projection where the basic circle is a simple close-packed plane such as (100) or (111).

1.18 The Wulff Net

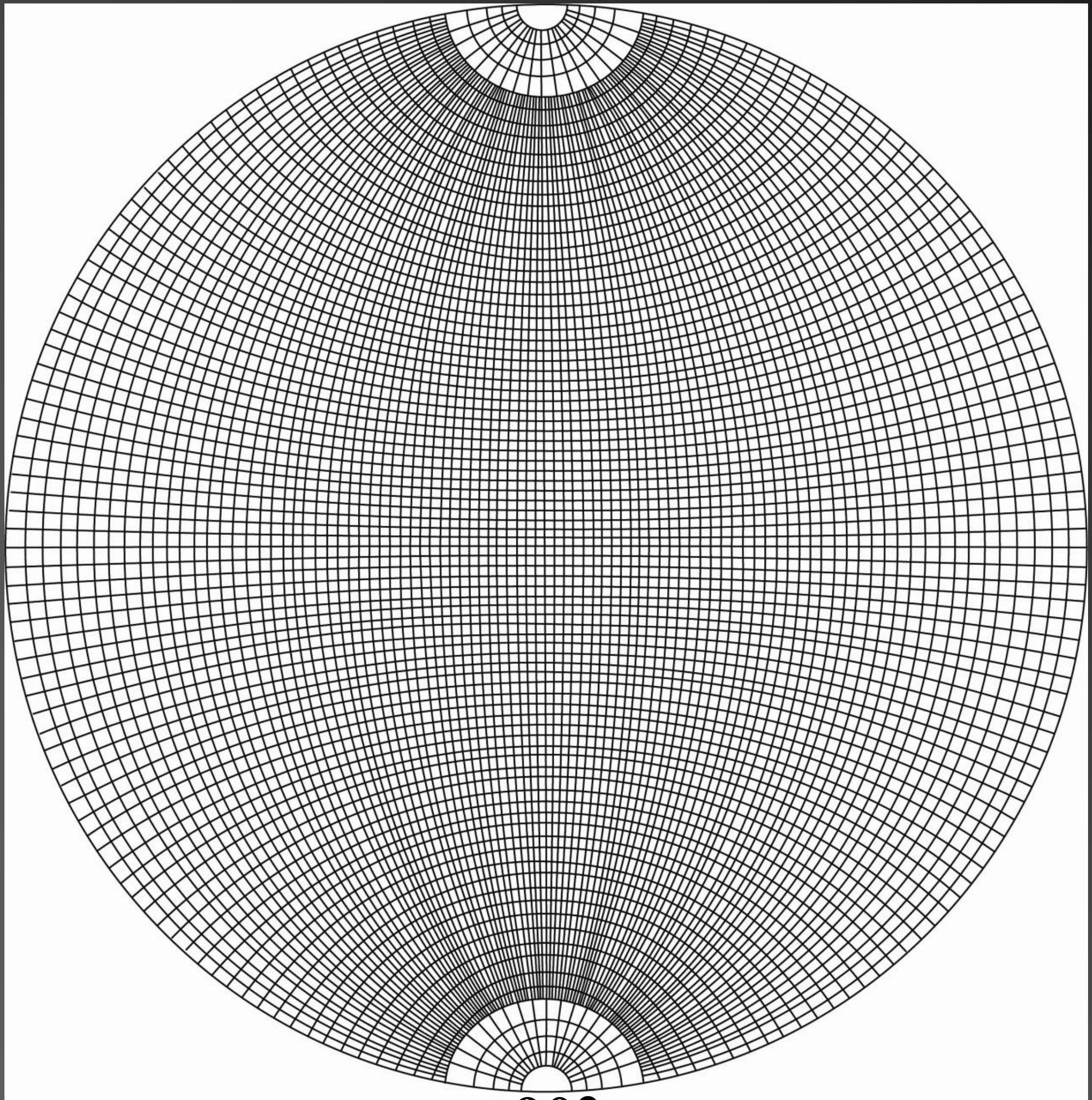


FIG. 1.26 Wulff, or meridional, stereographic net drawn with 2° intervals

Longitude

Latitude

2°

- The following two types of rotation of the plotted data are possible.

1. Rotation about an axis in the line of sight

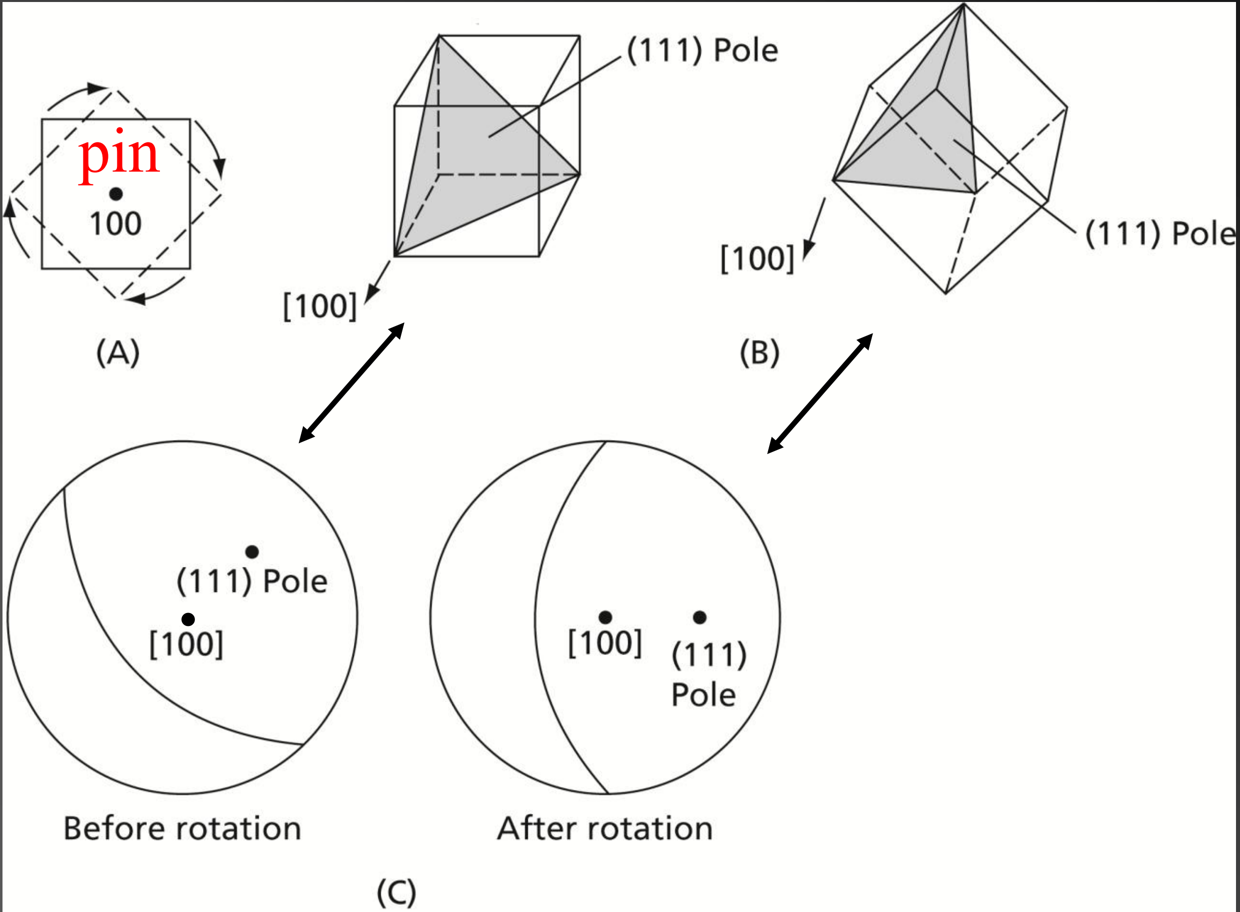


FIG. 1.27 Rotation about the center of the Wulff net. **(A)** The effect of the desired rotation on the cubic unit cell. Line of sight [100]. **(B)** Perspective view of the (111) plane before and after the rotation. **(C)** Stereographic projection of the (111) plane and its pole before and after rotation. Rotation clockwise 45° about the [100] direction

2. Rotation about the north-south axis of the Wulff net

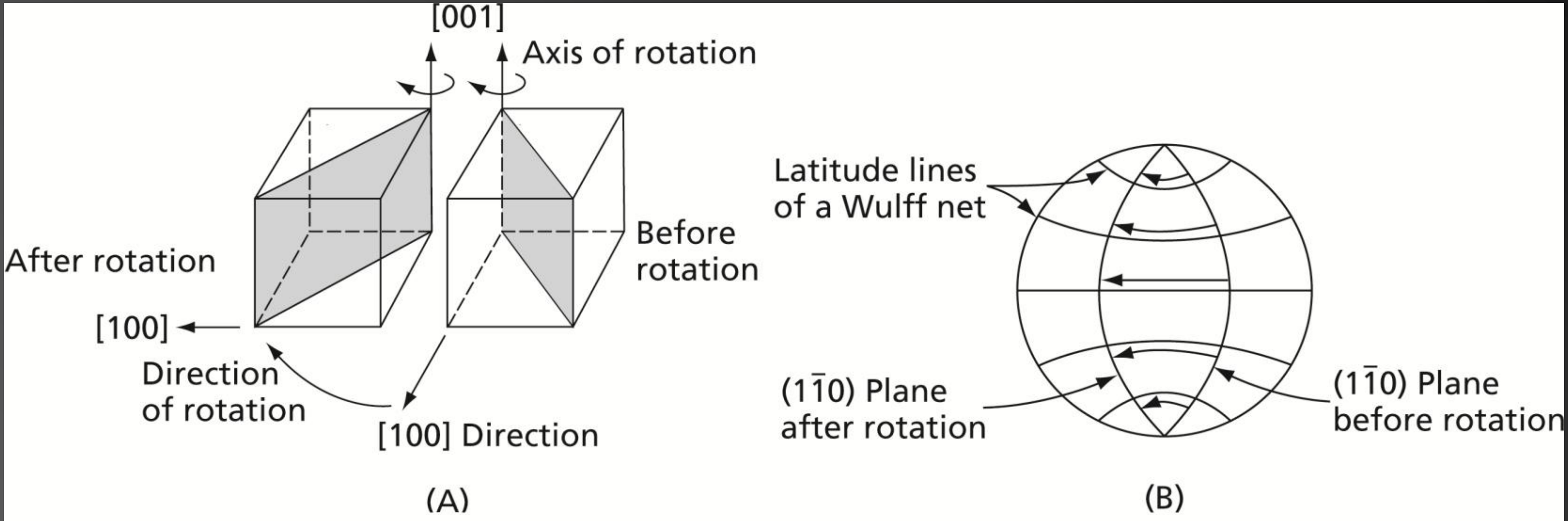


FIG. 1.28 Rotation about the north-south axis of the Wulff net. **(A)** Perspective views of the unit cell before and after the rotation showing the orientation of the $(1\bar{1}0)$ plane. **(B)** Stereographic projection showing the preceding rotation. For the sake of clarity of presentation, only the $(1\bar{1}0)$ plane is shown. The rotation of the pole is not shown. Also, the meridians of the Wulff net are omitted

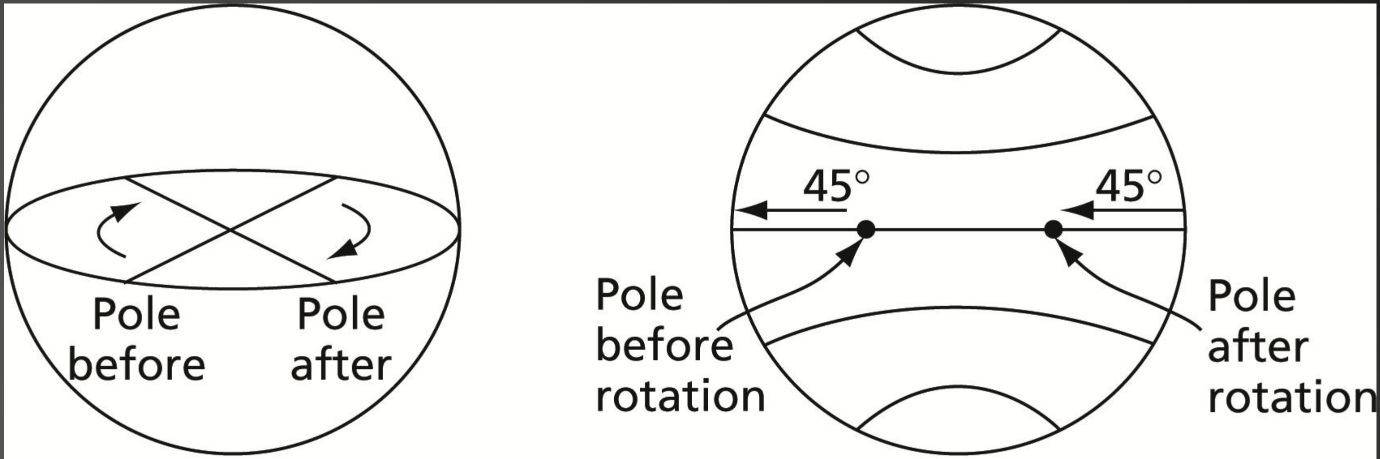


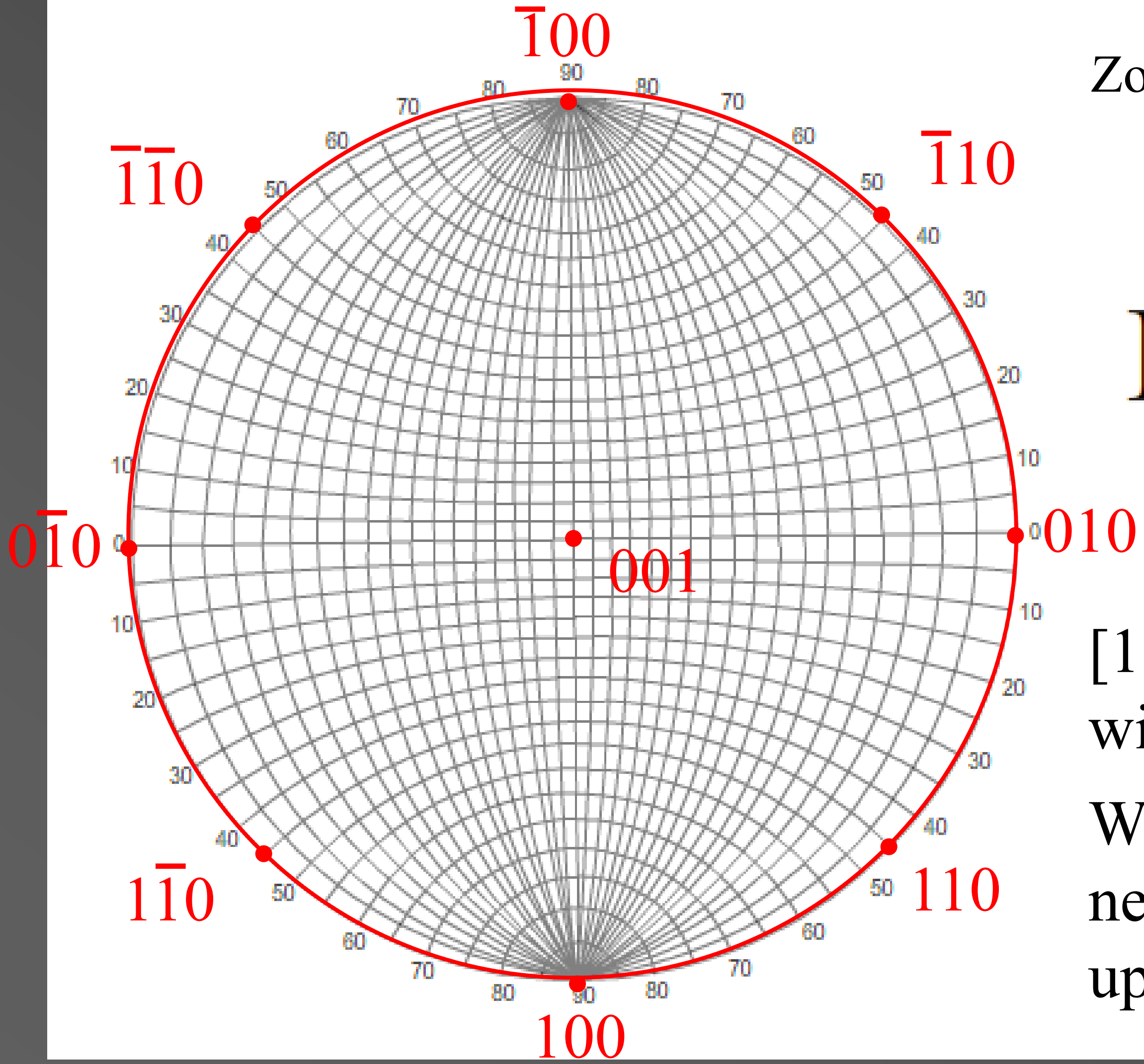
FIG. 1.29 The rotation of the pole of the $(1\bar{1}0)$ plane is given here. The diagram on the left shows the rotation in a perspective figure, whereas that on the right shows the motion of the pole along a latitude line of a stereographic projection which, in this case, is the equator

- **Not as simple to perform as 1.**

- to construct a stereogram

Family planes on the big circle. ⇒

1. Find zone axis on the big circle.
2. Find planes of a zone across the diameter on the big circle (90° away).
3. Rotate the Wulff net.

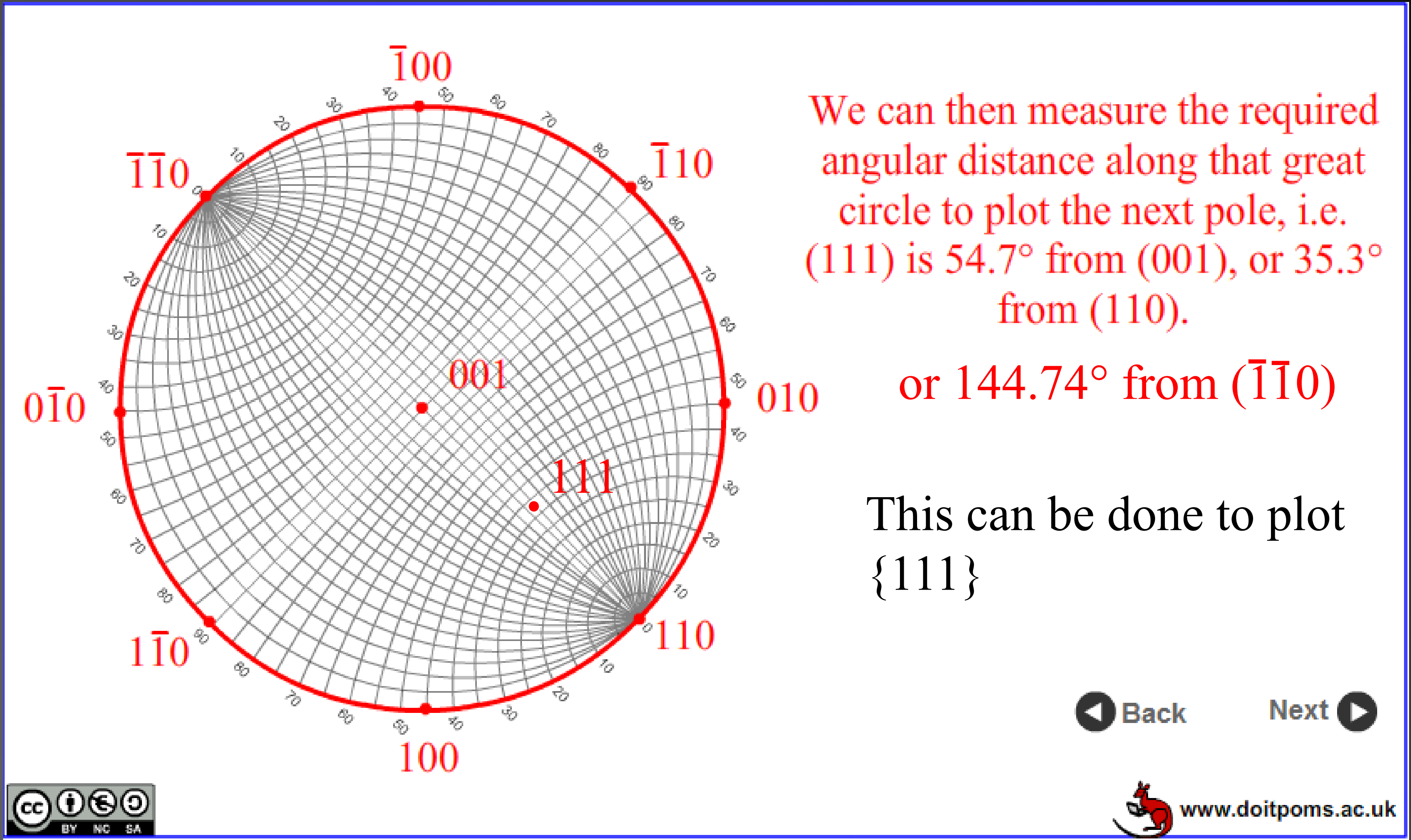


Zone axis of [111]: $\bar{1}10$ $1\bar{1}0$

Basics of the Wulff Net

[111] shares a zone axis with [001] and [110].

We need to rotate the Wulff net until a great circle lines up between them.



We can then measure the required angular distance along that great circle to plot the next pole, i.e. (111) is 54.7° from (001), or 35.3° from (110).

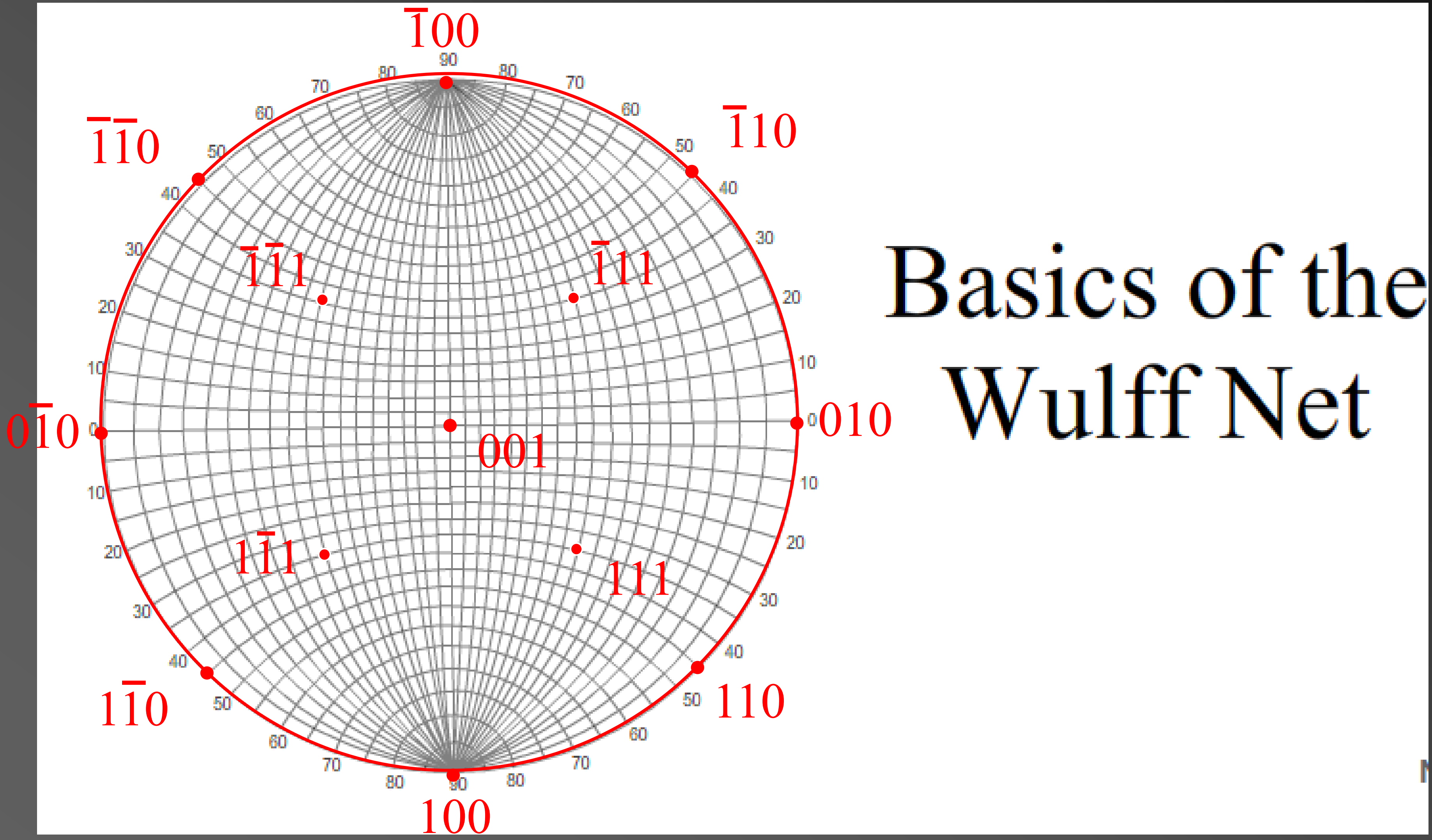
or 144.74° from ($\bar{1}\bar{1}0$)

This can be done to plot {111}

$[111] \bullet [001] \quad 1 = \sqrt{3}\cos\theta \quad \theta = 54.7$

$[111] \bullet [110] \quad 2 = \sqrt{6}\cos\theta \quad \theta = 35.37$

- to construct a stereogram



Basics of the Wulff Net

1.19 Standard projections

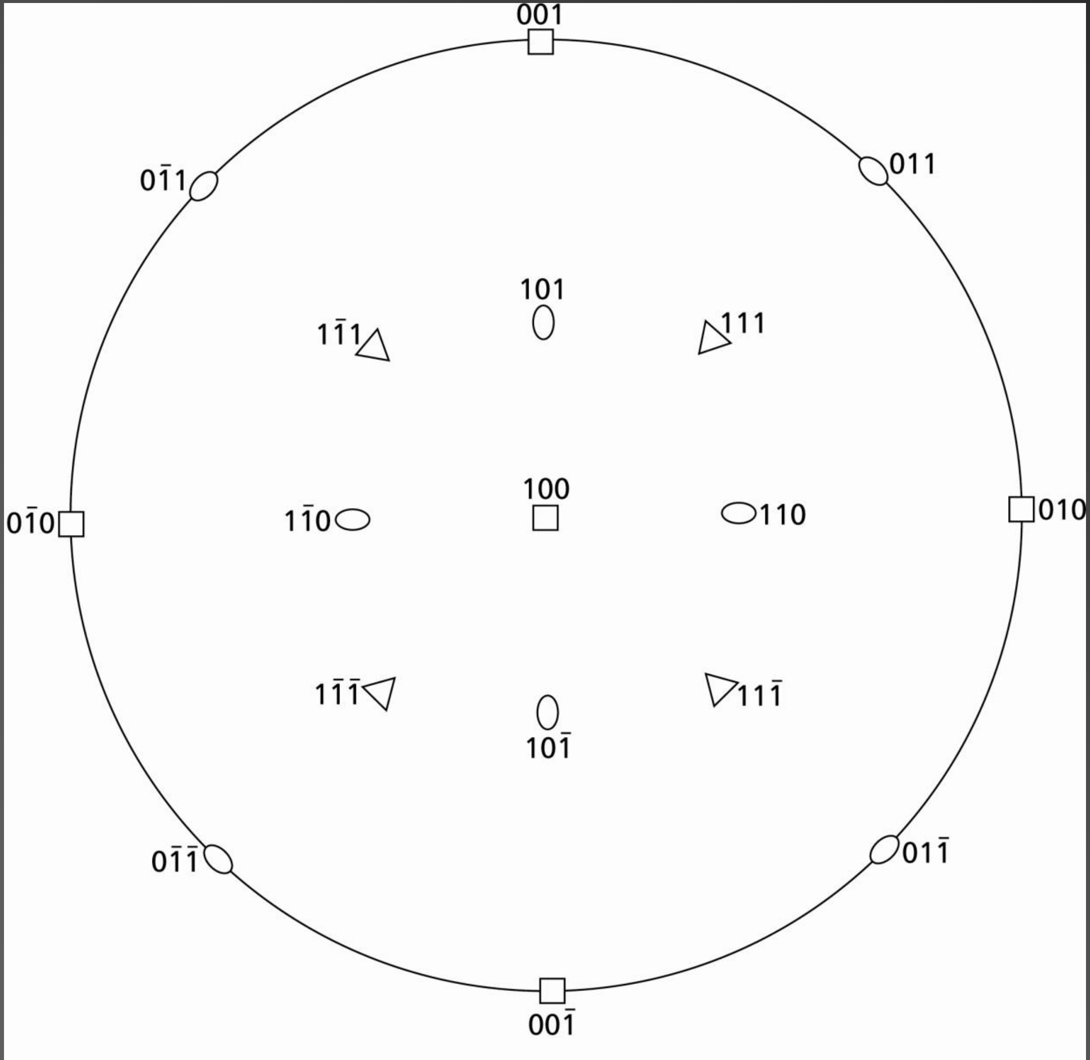
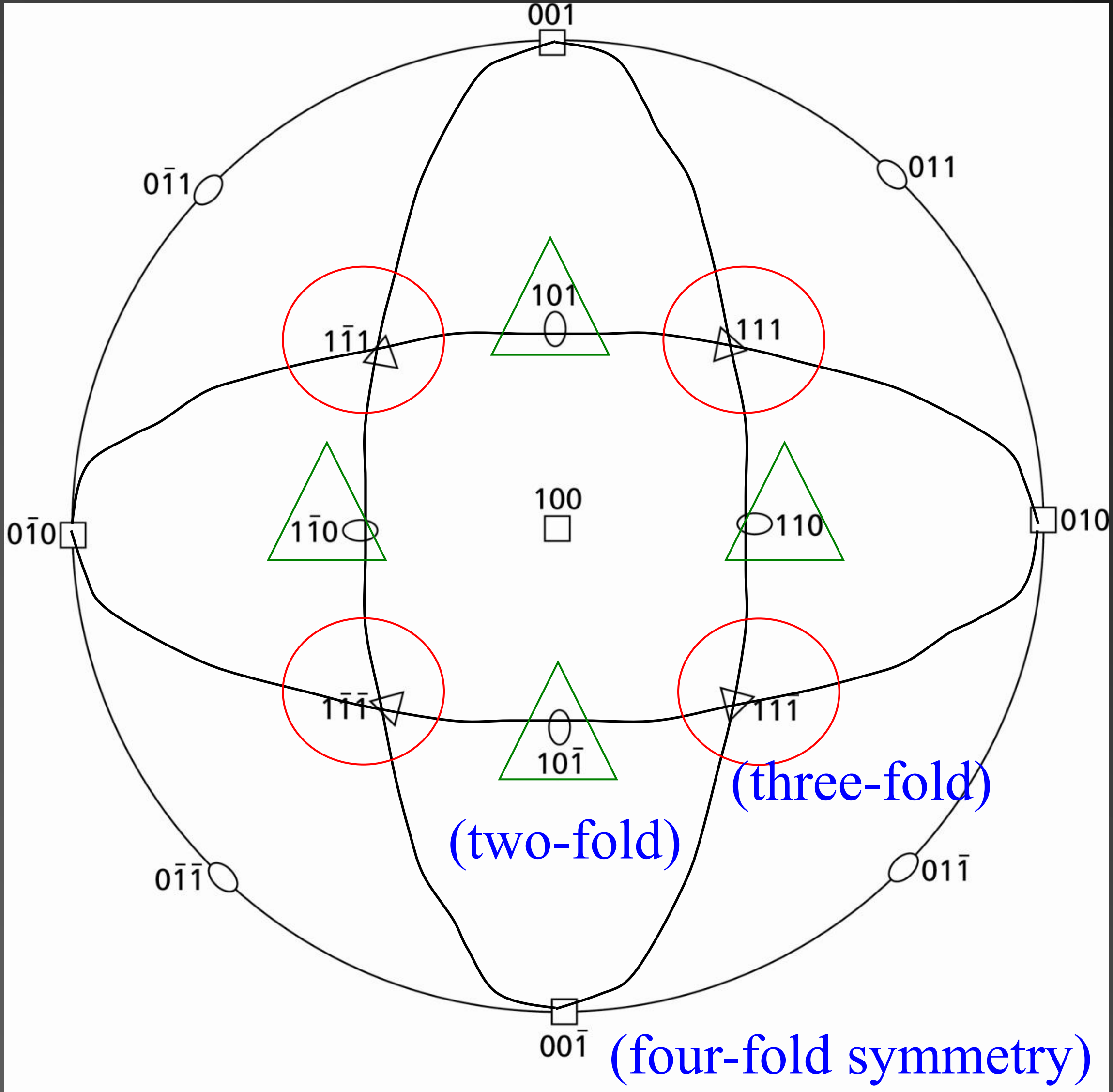


FIG. 1.30 A 100 standard stereographic projection of a cubic crystal

- (100) pole normal to the screen
- {100}: squares: four-fold symmetry axes (90°)
- <111>: triangles: three-fold symmetry (120°)
- <110>: ellipses: two-fold symmetry (180°)



(three-fold)

(two-fold)

(four-fold symmetry)

FIG. 1.30 A 100 standard stereographic projection of a cubic crystal

More complete 100 standard projection

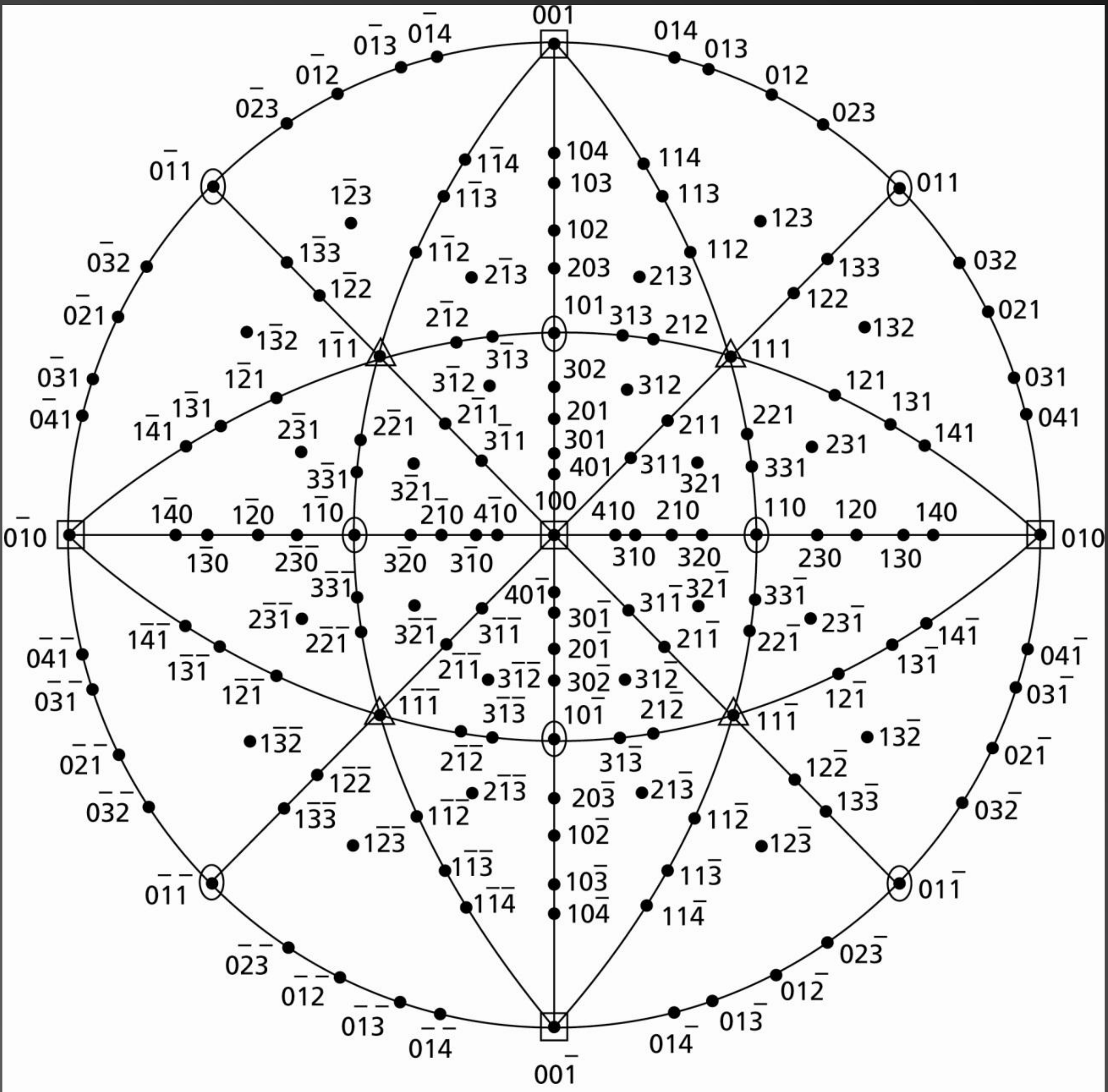


FIG. 1.31 A 100 standard stereographic projection of a cubic crystal showing additional poles

More complete 111 standard projection

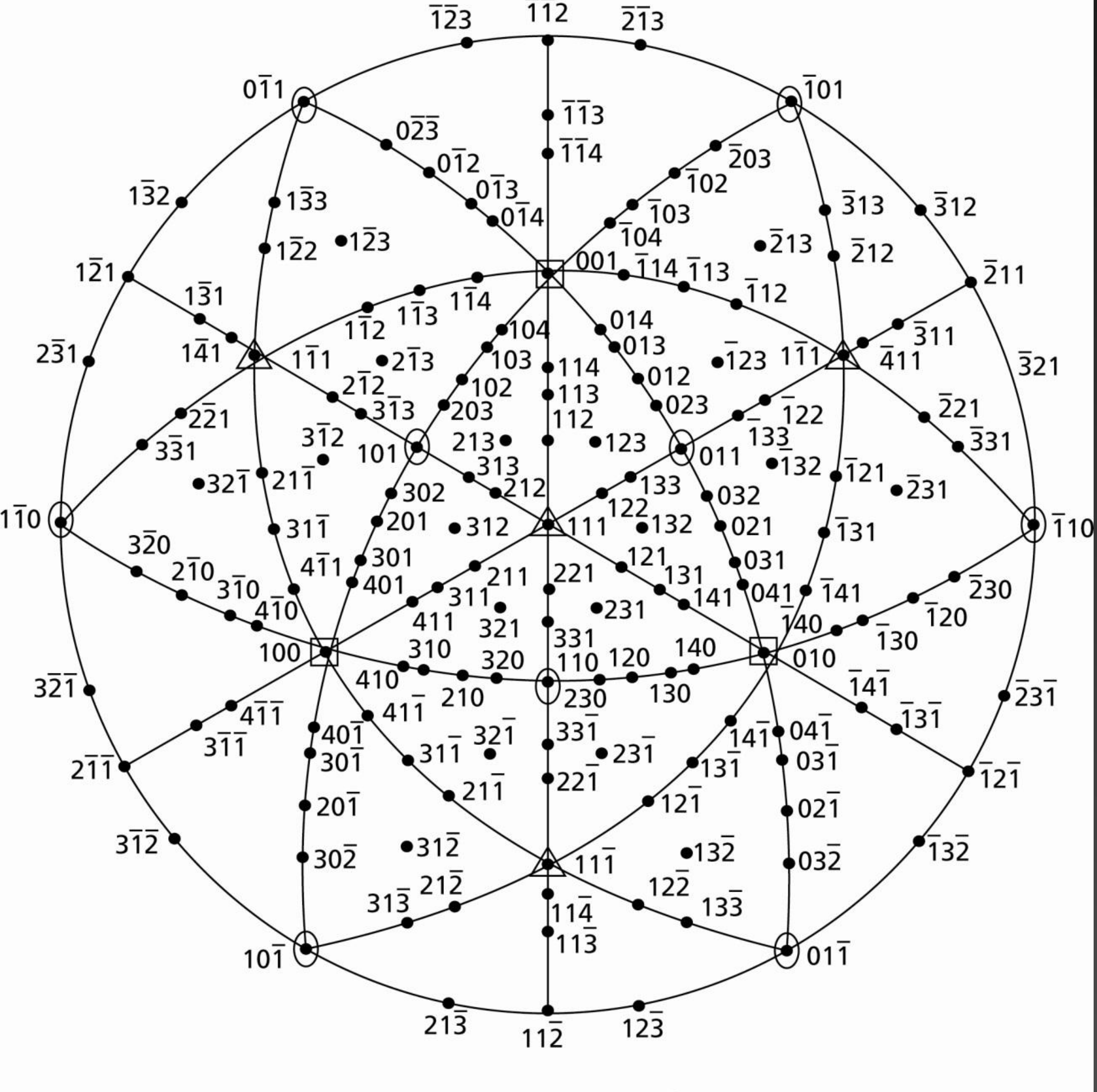


FIG. 1.32 A 111 standard projection of a cubic crystal

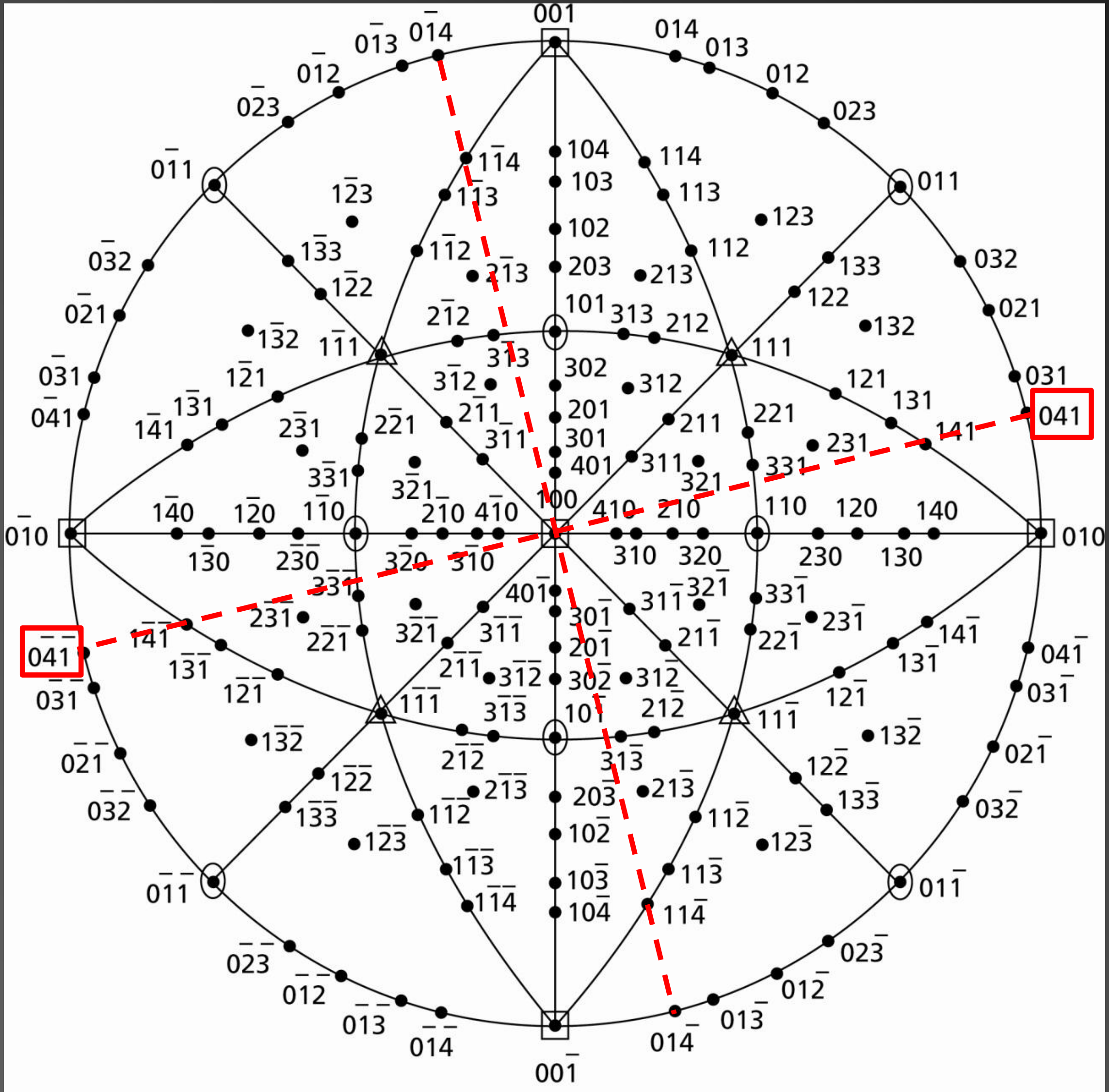


FIG. 1.31 A 100 standard stereographic projection of a cubic crystal showing additional poles

Q: identify (1-14)

poles (041)

(0-4-1)

Planes of a zone

1.20 The standard stereographic triangle for cubic crystals

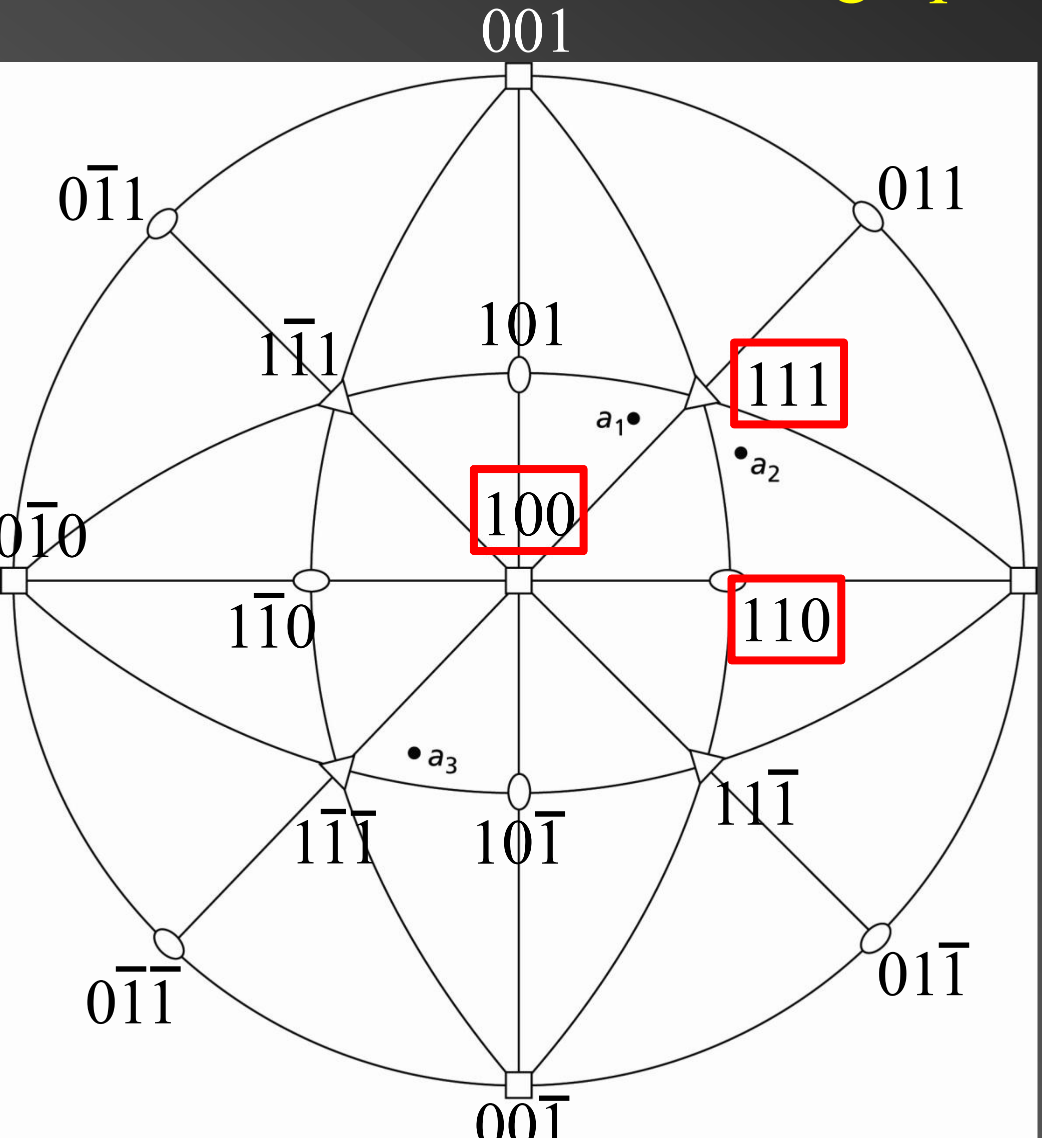


FIG. 1.33 The crystallographic directions a_1, a_2, and a_3 shown in this standard projection are equivalent because they lie in similar positions inside their respective standard stereographic triangles

- there are 24 similar triangles (24 similar triangles in the rear)
- The triangles are formed by **a <111>, a <110>, and a <100>**
- **each triangle corresponds to a region of the crystal that is equivalent.**

- a1, a2, a3 are crystallographically equivalent because they are located at the same positions inside three triangles.

  ⇒ Cut three samples, with axes parallel to a1, a2, and a3, out of a large single crystal.

  ⇒ ex. stress-strain curve, electrical resistivity are identical.

- The plotting of crystallographic data is often simplified because of the equivalence of the stereographic triangles.

- If one has a large number of long, cylindrical crystals and wishes to plot the orientations of the individual crystal axes, this can be done conveniently in a single stereographic triangle.

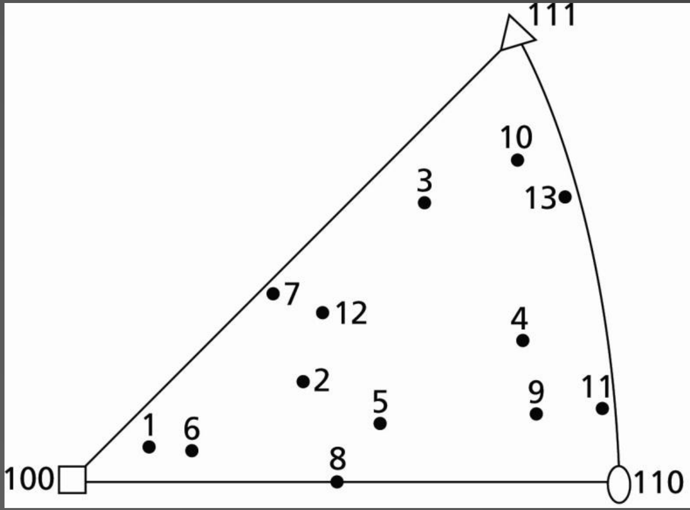


FIG. 1.34 When it is necessary to compare the orientations of a number of crystals, this often can be done conveniently by plotting the crystal axes in a single stereographic triangle, as indicated in this figure