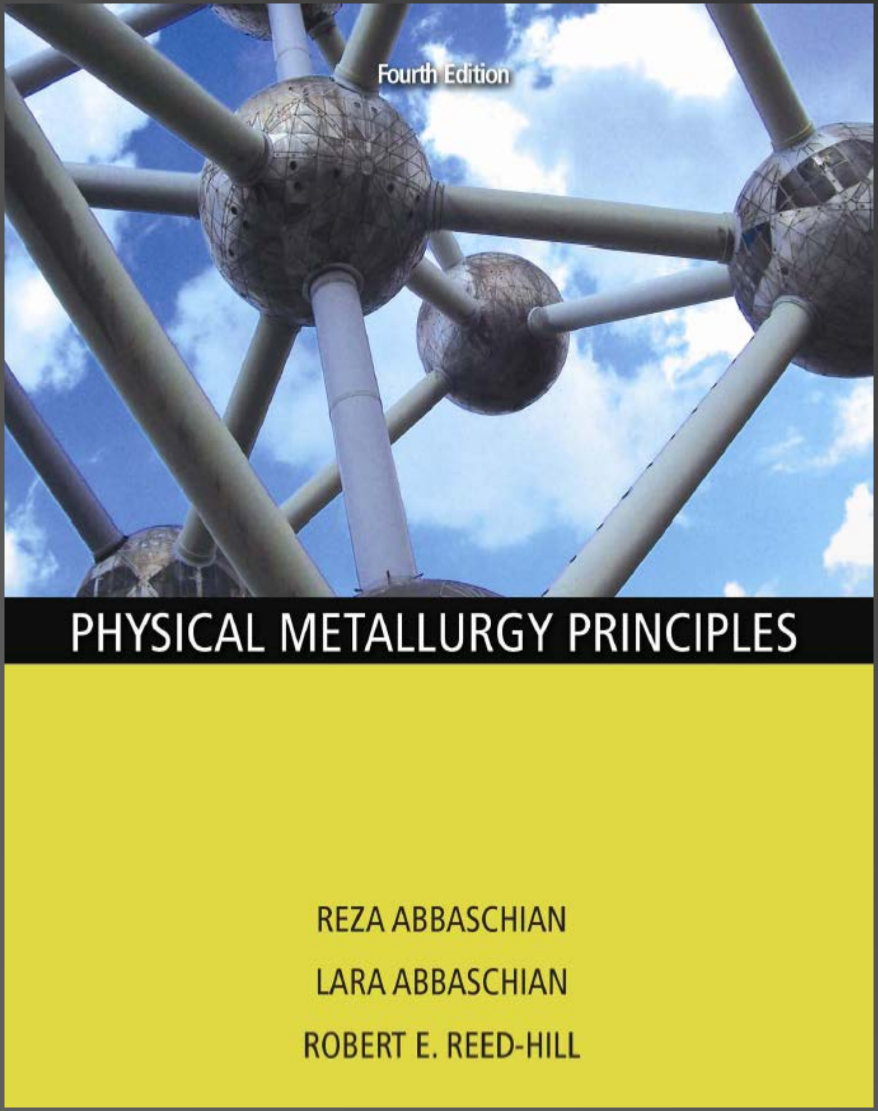
Physical Metallurgy Principles

Fourth

SI Version



Fourth Edition

PHYSICAL METALLURGY PRINCIPLES

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Chapter Five:

Dislocations and Plastic Deformations

Connected with the relation of dislocations to plastic deformation

Imperfections

1. Point defects: vacancy atoms, substitutional and interstitial atoms (Ch. 7^{th})
2. Line defects: dislocations (Ch. 4th and 5th)
3. Area defects: grain boundaries (Ch. 6)

- Dislocations must be created during **plastic deformation.**
- There are a number of ways that dislocations may be created as a result of plastic deformation.
- **Frank-Read source** is one of considerable significance.

5.1 The Frank-Read Source (generator)

- An edge dislocation ($\overline{xy}$) on plane ABCD is connected to two other edge dislocations.
- Under the applied stress τ (top & bottom) => only the edge dislocation lying on the slip plane ABCD could move.
- The other two sections of dislocations can move by applied stress **only on front and back** by the red arrows shear stress.

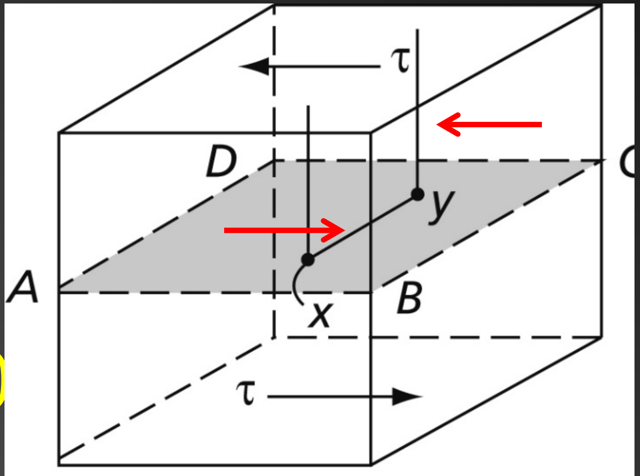
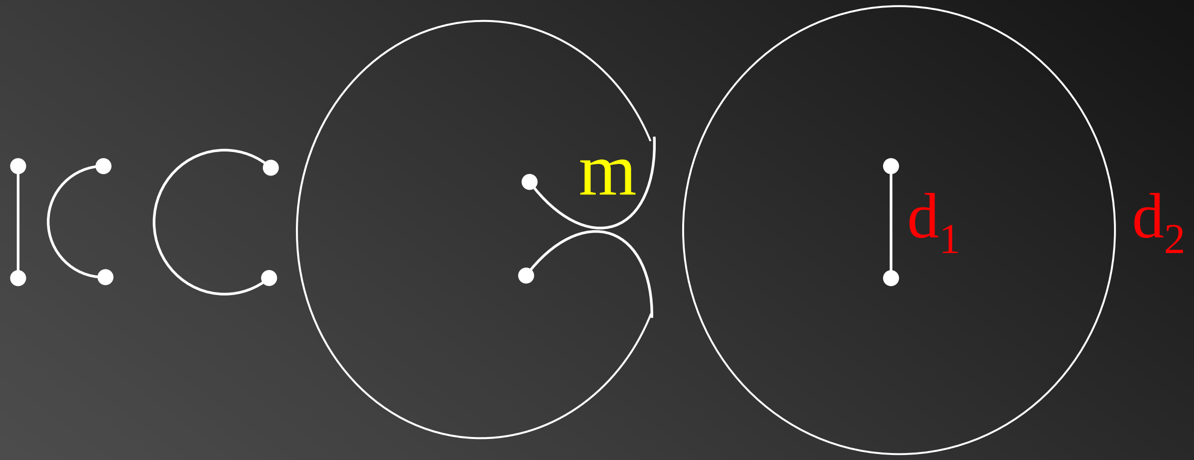
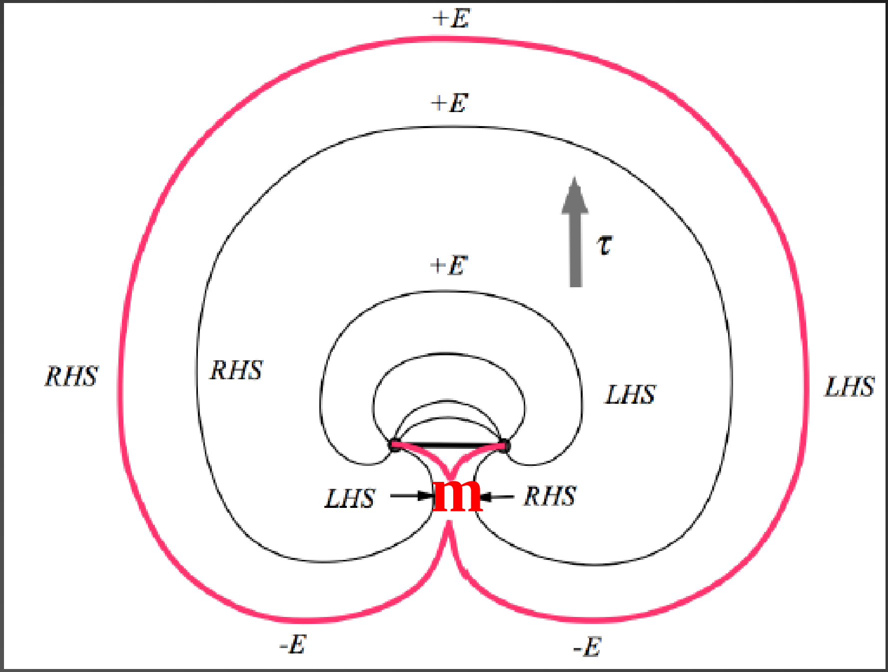


FIG. 5.1 Frank-Read source. The dislocation segment *xy* may move in plane *ABCD* under the applied stress. Its ends, *x* and *y*, however, are fixed





- Left & right screw intersects at m => cancellation
- Break into two: segment d_1 and loop d_2 (expanding to the surface of the crystal.)
- This process could continue and generate a lot of loops on the same slip plane.
- Frank-Read source: dislocation generator.

http://en.wikipedia.org/wiki/Frank-Read_Source

- As long as the newly formed loops are able to expand and thus move away, the Frank-Read source can create **an infinite number of dislocation loops**.

- If one of the leading dislocation loops is stopped by a grain boundary, for an example, a back stress **builds up** at the source, and eventually its output ceases.

  ⇒ or the dislocations behind the stalled dislocation will **pile up**.

- A variant of the Frank-Read mechanism was proposed by Koehler. He pointed out that a dislocation segment that has **a screw orientation** can move from its slip plane on to an intersecting slip plane: **cross slip**.

Elementary Dislocation Theory, J. Weertman and J. R. Weertman, P. 125.

5.2 Nucleation of Dislocations

- Dislocations can also be formed without the aid of Frank-Read or similar sources.

- Dislocations can also be created by a nucleation process

  (1) homogeneous: formed in a perfect lattice only by the action of a stress, **no agency other than stress being required**.

  ⇒ requires **extremely high stresses**

  ⇒ For typical metal this value ~ 10 GPa

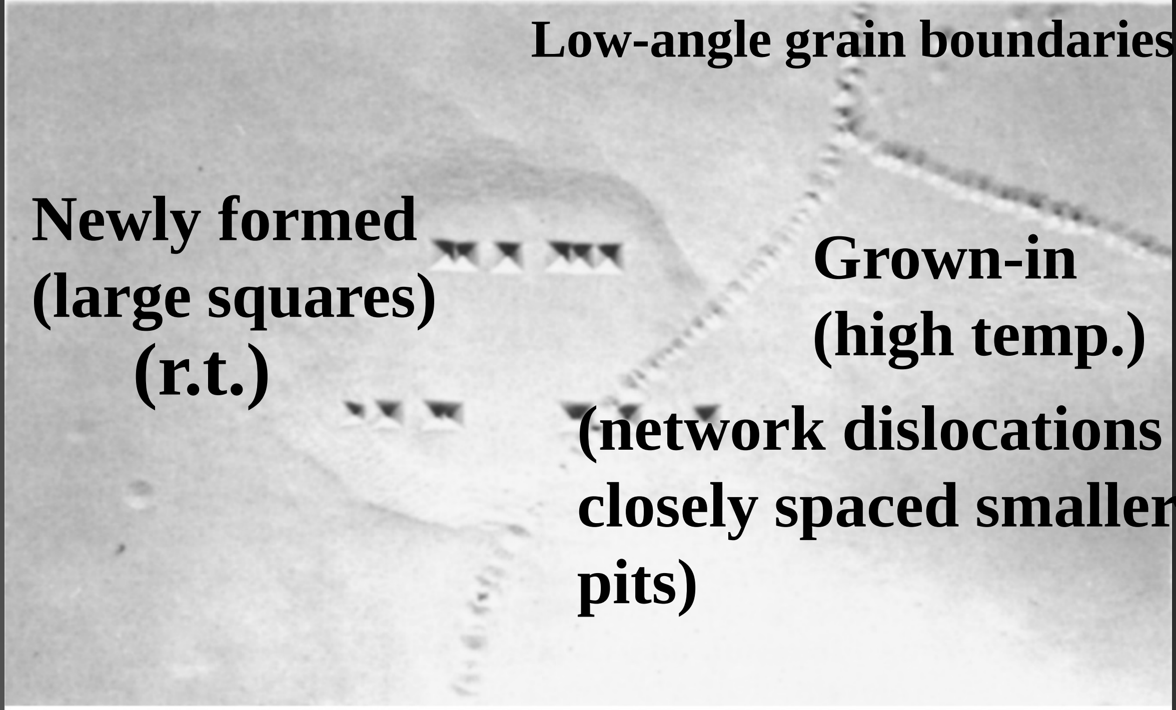
  (2) heterogeneous: formed with the help of defects (impurities, precipitate)

  ⇒ The defects make the formation of dislocations easier by lowering the applied σ required to form dislocations

  ⇒ typically observed value ~ several MPa (Mg: 0.7 MPa)

- If dislocations are not formed by F-R, then they must be nucleated heterogeneously.

- Metal is not suitable for the dislocation nucleation studies
  ⇒ Because even very small deformations can produce relatively high dislocation density: difficult to observe the motions of individual dislocations.
- LiF crystals (simple cubic) are able to introduce **controlled, small numbers** of dislocations: quantitatively observe the movements of both screw and edge dislocations.
- One of the best and simplest ways of observing dislocations: an etching reagent: etch pit.
- Several etching solutions have been developed for use in **LiF.**
  1. One of which is capable of distinguishing between grown-in (when manufactured) and newly formed dislocations.



Low-angle grain boundaries

Newly formed
(large squares)
(r.t.)

Grown-in
(high temp.)

(network dislocations
closely spaced smaller
pits)

FIG. 5.3 The large square etched pits in horizontal rows correspond to dislocations formed in LiF at room temperature, while the smaller, closely spaced pits lying in curved rows were grown into the crystal when it was manufactured (Gilman, J. J., and Johnson, W. G., *Dislocations and Mechanical Properties of Crystals*, p. 116, John Wiley and Sons, Inc., New York, 1957. Used by permission of the author.)

- One observation: distinguishing between grown-in (H.T.) and newly formed dislocations. Possible reasons:
  => Dislocations formed at high temp. are more likely to have impurities segregated around them than dislocations created in the lattice at room temp.
  ⇒ Higher temp. impurity diffusion quicker.

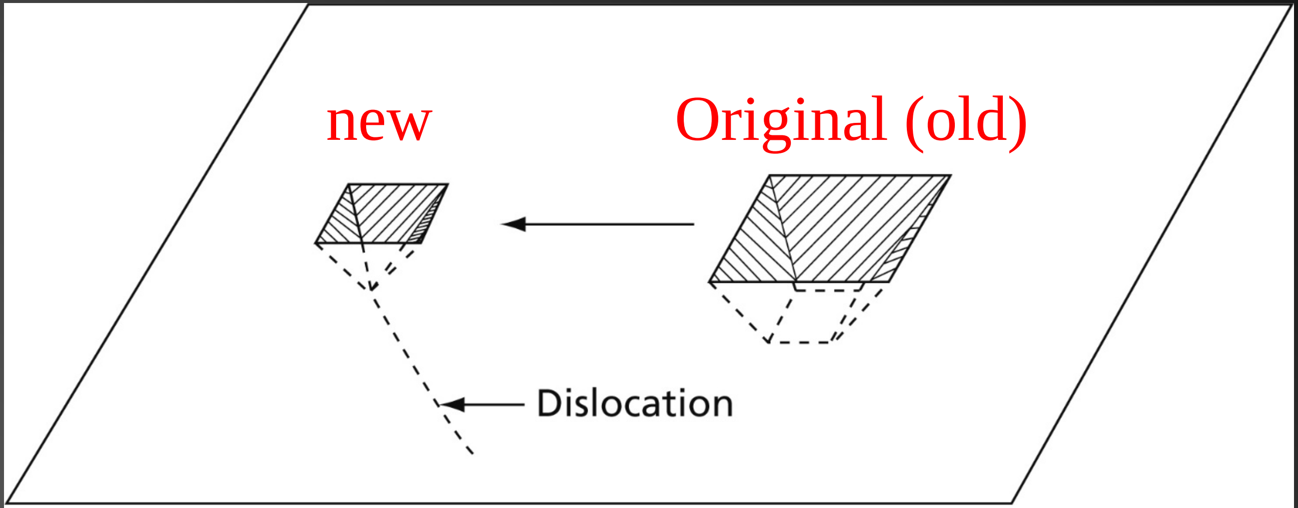
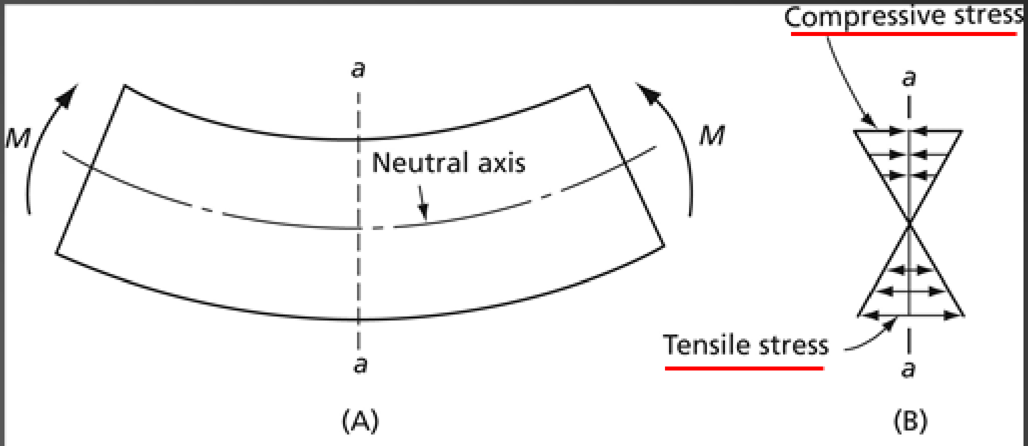


FIG. 5.4 Dislocation movement in LiF as revealed by repeated etching (Reprinted with permission from J.J. Gilman and W.G. Johnson, *Journal of Applied Physics*, Vol. 30, Issue 2, Page 129, Copyright 1959, American Institute of Physics)

2. Another interesting facet: one can follow the movement of a dislocation under the action of an applied stress.
   => The pits actually connected with the dislocation always **have pointed extremities**.
   => While those from which the dislocations have moved have flat bottoms.

5.3 Bend Gliding:

- Crystals could be plastically bent by the mechanism of slip. The bending can be explained in terms of F-R or other sources.



Max.

zero

Max.

FIG. 5.5 (A) The elastic deformation of a crystal subject to two equal moments (*M*) applied at its ends. **(B)** The normal stress distribution on a cross-section such as *aa*

- The stress distribution: $\sigma_x = \dfrac{My}{I}$

M: bending moment
y: vertical distance
I: moment of inertia
(= $\pi r^4/4$; for a circular rod)

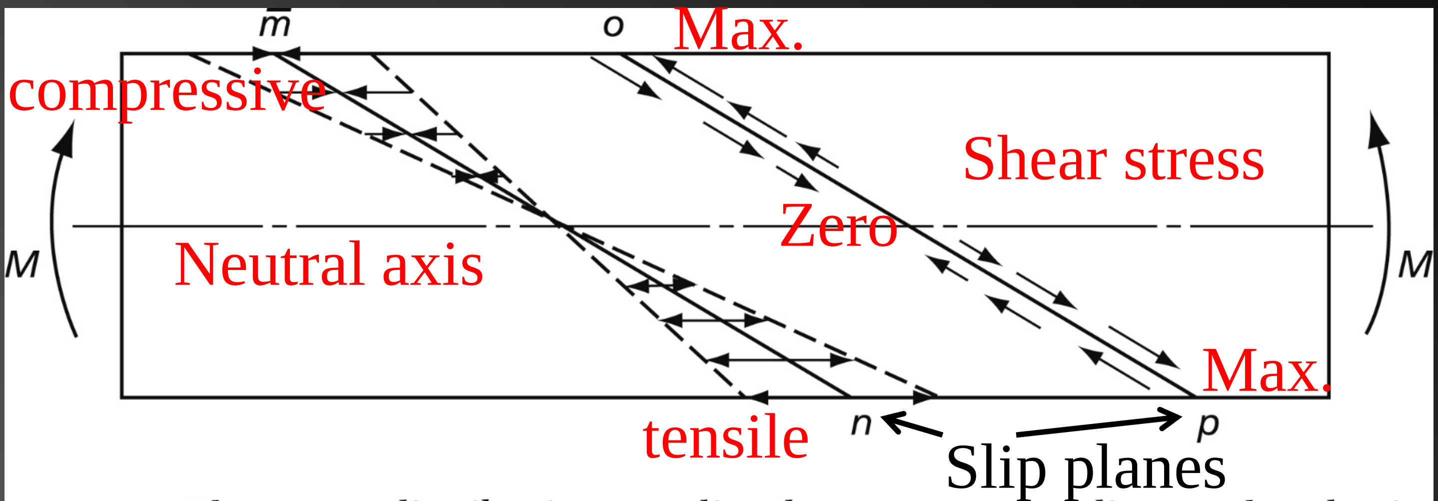


FIG. 5.6 The stress distribution on slip planes corresponding to the elastic deformation shown in Fig. 5.5

- Line mn is assumed to represent the trace of a slip plane, and ⊥ the plane of the paper, and mn is also the slip direction.
- Line op, the shear-stress component, sign changes across the neutral axis.
- The first dislocation loops will form at Frank-Read or other sources close to either the upper or lower surface (maximum stresses).

  ⇒ Dislocation loop is getting bigger

  ⇒ **The loop is free to move. It will encounter other dislocations, and become part of a network.**

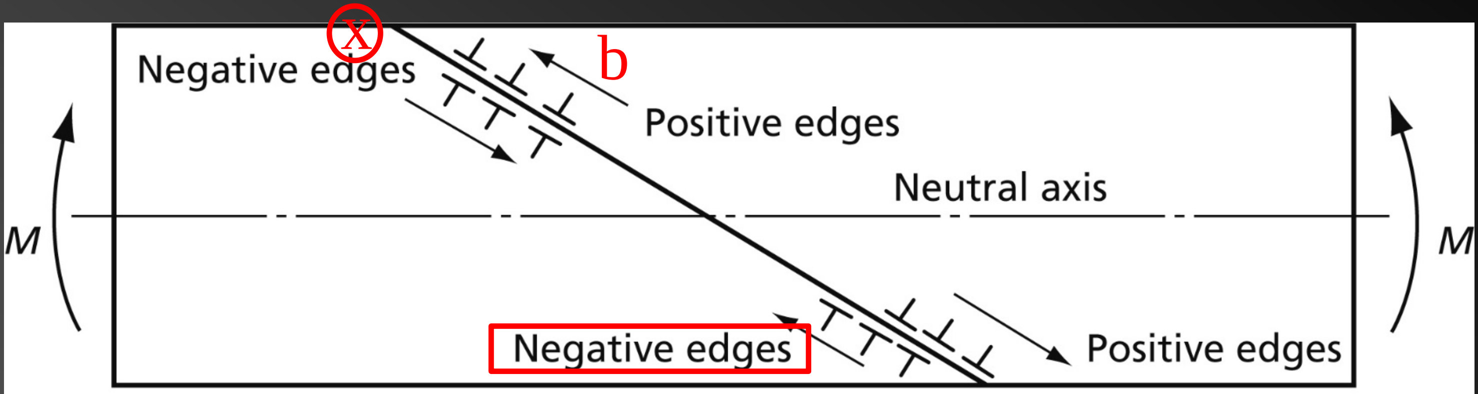
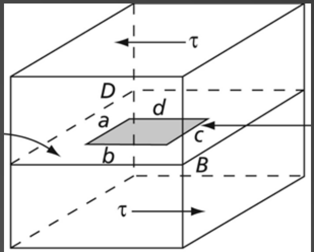


FIG. 5.7 The effect of the stress distribution on the movement of dislocations. Positive-edge components move toward the surface; negative edges toward the neutral axis

- The positive edge components of all dislocation loops move toward the upper or lower surface. => high stress
- Right- and left-hand screw components => either into or out of the paper.

  => all these three components move to the surface and leave the crystal.

a: positive edge dislocation
c: negative edge dislocation
b: right-hand screw
d: left-hand screw



- The negative edge components move toward the neutral axis. => decreasing shear stress => eventually stop.

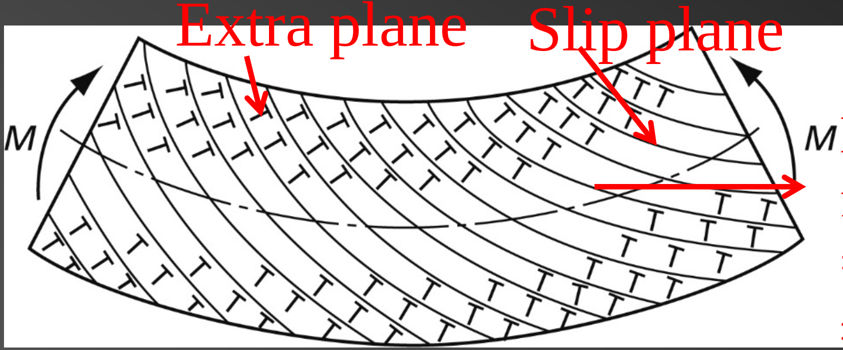


FIG. 5.8 Distribution of the excess edge dislocations in a plastically bent crystal

Narrow section surrounding the axis:
free of dislocations
=> under moderate stresses (not plastic)
⇒ will not be stressed above the elastic limit

- Frank-Read sources eventually become negative-edge dislocations that move toward the neutral axis.
- Bent further, negative-edge dislocations driven further along the slip plane toward the neutral axis.
- Eventually an orderly sequence of dislocations, uniform separation, minimum spacing (same type, same sign mutually repel).

- Bending reversed, positive, slip plane curvature reversed.
- Not limited to large crystals; may occur in quite small crystals, or even in extremely small areas of crystals.

5.4 Rotational Slip:

- A third type of deformation besides shear and bending

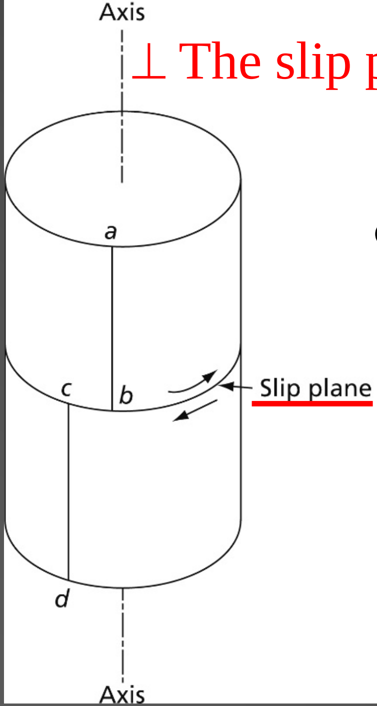


FIG. 5.9 A single crystal can be rotated about an axis normal to a slip plane that contains several slip directions

⊥ The slip plane

- Torsional deformation such as this can be explained in terms of screw dislocations lying on the slip plane.

- Unlike bending, more than one set of dislocations required.
  => The slip plane must contain more than one possible
  slip direction.
- Hexagonal basal plane & FCC {111} planes are ideal for this deformation.

Atoms just below the slip plane.

Atoms just above the slip plane.

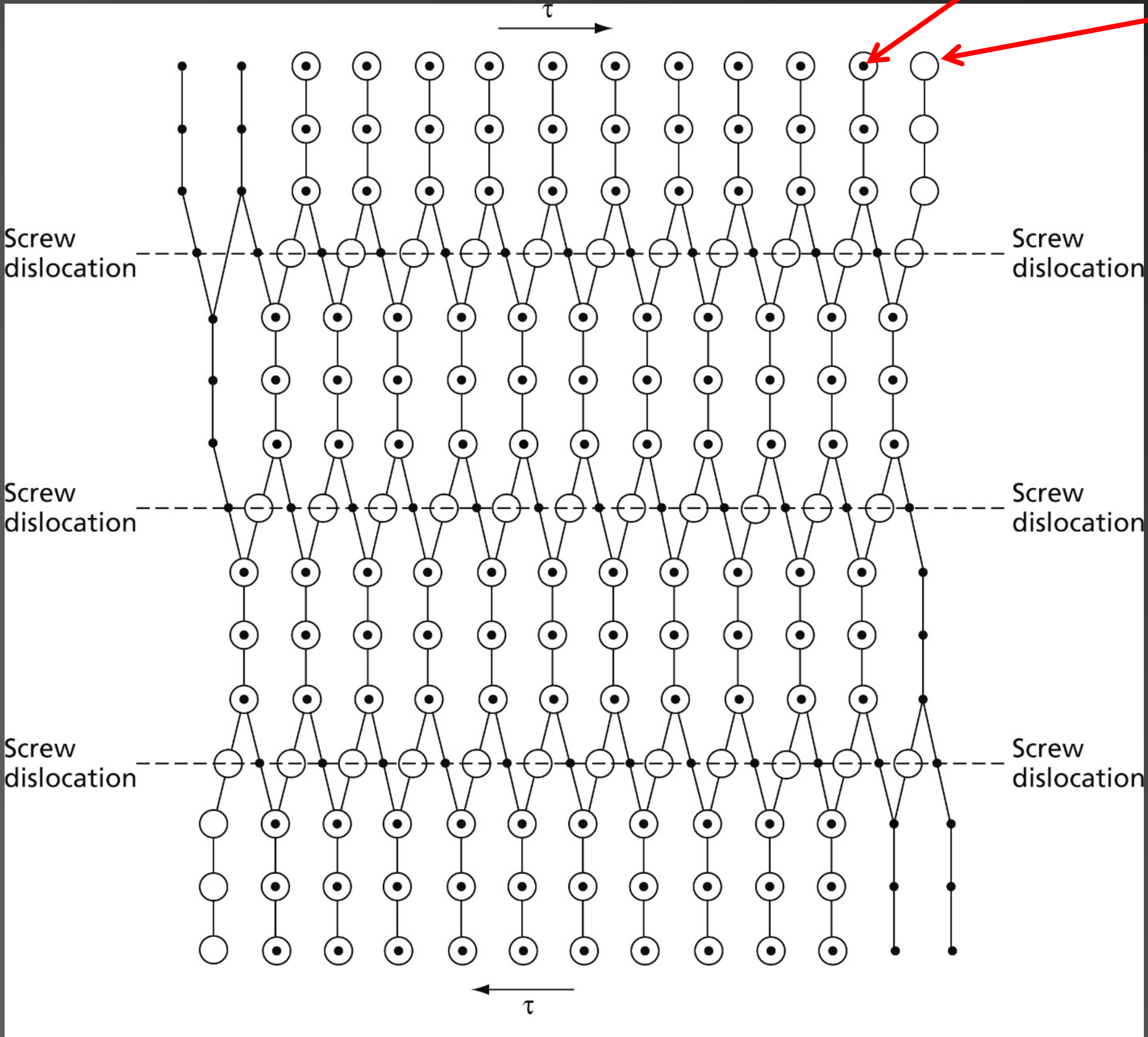


FIG. 5.10 An array of parallel screw dislocations. Open circles represent atoms just above the slip plane, while dots correspond to atoms just below it

- A single array of parallel screw dislocations.
- Such a dislocation arrangement shears the material above and below the **slip plane** in the **horizontal** direction.
- Strain fields are high
- The individual parallel dislocations are additive.

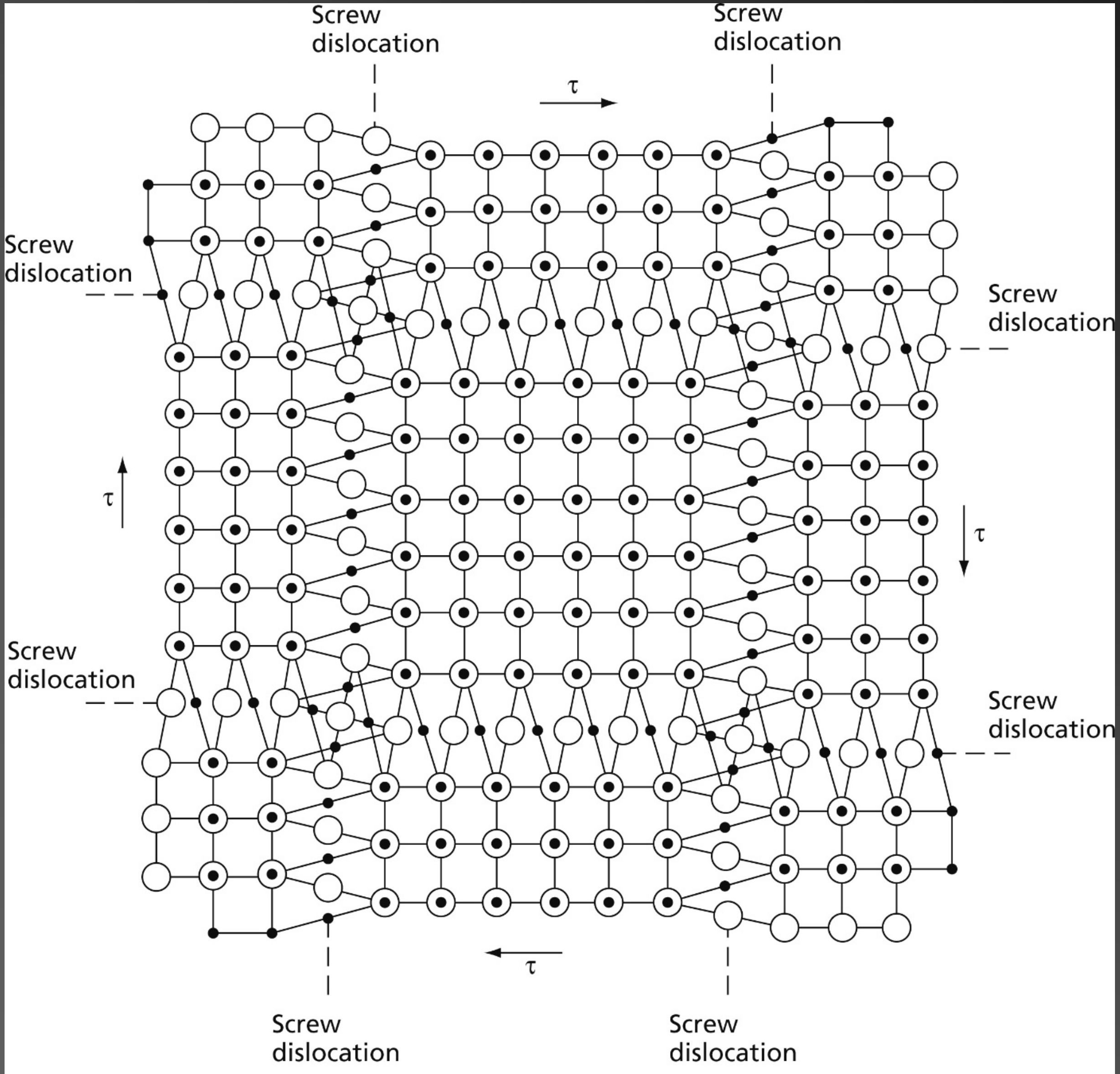


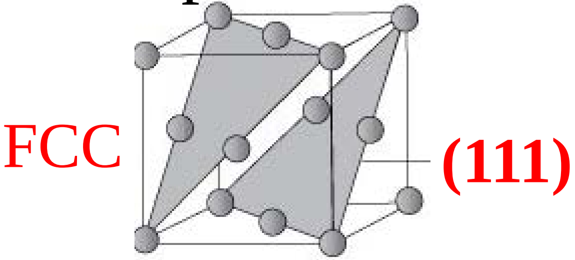
FIG. 5.11 A double array of screw dislocations. This array does not have a long-range strain field; open circles show atoms above the slip plane, while dots represent those below the plane

- For as true rotation, one needs a similar component of **shear at 90° to this direction.**
- Strain fields: small
  => compensate each other.
  => **Low strain energy**.
- The amount of slip deformation by this mechanism can be very **large**.
  => Ex. > 1 cm dia. (about its basal plane pole): 10 revolutions/inch for Zn
  ⇒ on many slip planes.

5.5 Slip Planes and Slip Directions

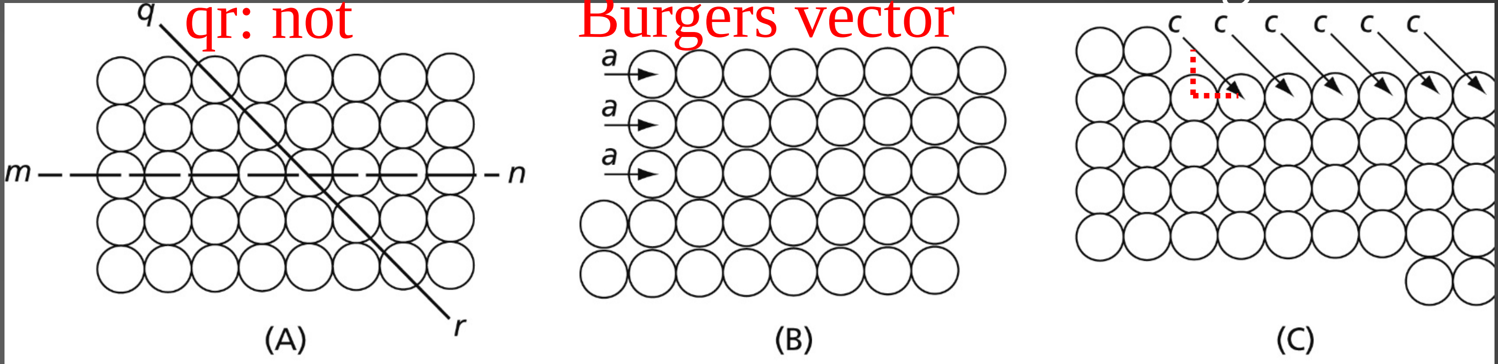
- Experimental fact: slip occurs preferentially on planes of **high atomic density**
- Separation between planes is proportional to the density of the planes. (**D↑=> d↑**)

  ⇒ the separation between close-packed planes is greater than for other crystal planes: e.g. (111) most close packed plane => **wide plane – plane distance.**



- Crystals are sheared most easily on planes of wide separation (slip planes).
- Dislocations move more easily along planes of wide spacing where the lattice distortion due to the movement of the dislocation is small.

- Slip direction (shear direction) has been found to be exclusively a **close packed direction**.
- **Even much stronger than slip plane.**
- When a crystal is sheared by an amount equal to the Burgers vector of the dislocation, the symmetry of the crystal must be retained.
  => The smallest shear that can fulfill this condition is a close packed direction.



mn: close packed direction

qr: not

Burgers vector

Burgers vector

FIG. 5.12 Two ways in which a simple cubic lattice can be sheared while still maintaining the lattice symmetry: **(A)** Crystal before shearing, **(B)** shear in a close-packed direction, and **(C)** shear in a non-close-packed direction screw

$$W_s = \frac{\mu b^2}{4\pi}\ln\left(\frac{r'}{r_0}\right)$$

edge

$$W_s = \frac{\mu b^2}{4\pi(1-\nu)}\ln\frac{r'}{r_0}$$

- Strain energy $w = (\mu b^2/4\pi)\ln(r'/r)$
- $c^2 = 2\ a^2$, c^2 twice than a^2
- Close packed direction: smallest strain energy.
  => does not disturb the crystal structure.
  => more probable than forms of higher strain energy.

mn: close packed direction

qr: not

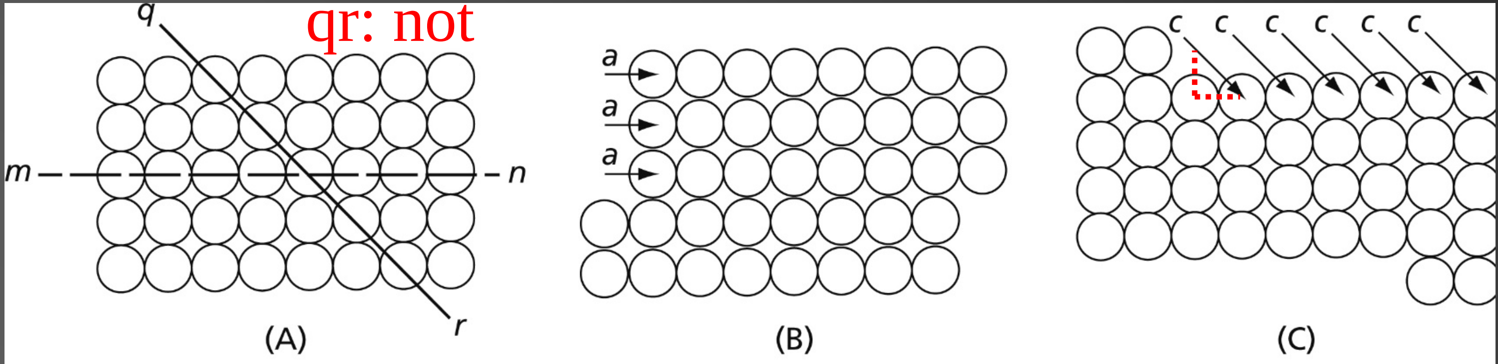


FIG. 5.12 Two ways in which a simple cubic lattice can be sheared while still maintaining the lattice symmetry: **(A)** Crystal before shearing, **(B)** shear in a close-packed direction, and **(C)** shear in a non-close-packed direction

5.6 Slip Systems:

- Slip planes + Slip directions => slip systems (modes)
- FCC slip system: major operative slip system **<110> {111}**
- BCC slip system: the slip direction is almost always <111>; the most densely packed plane are **{110},**
- HCP slip systems: slip directions: **<11-20>**; slip planes: **(0001)**

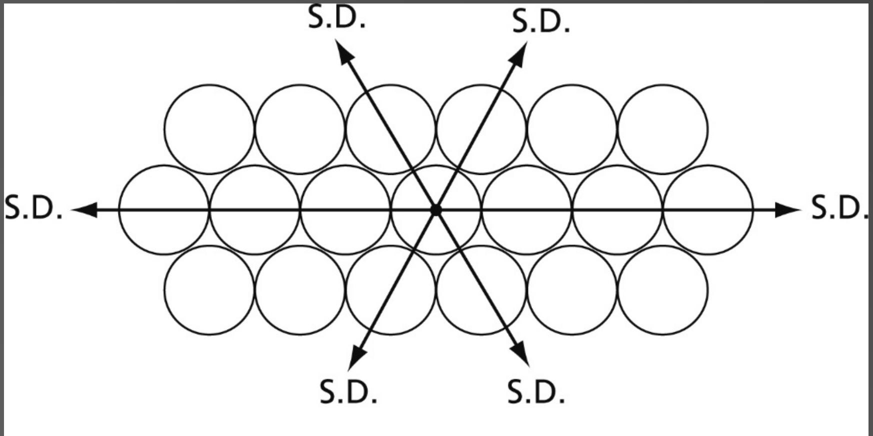


FIG. 5.13 The three slip directions (S.D.) in a plane of closest packing. Notice that this type of plane occurs in both the hexagonal close-packed and the face-centered cubic lattices

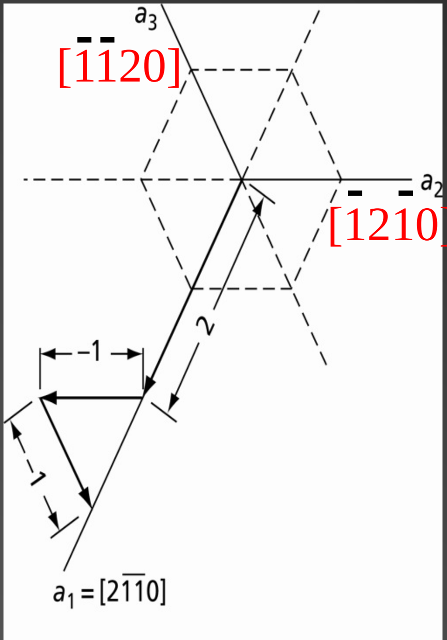


FIG. 1.18 Determination of indices of a digonal axis of Type I—$[2\bar{1}\bar{1}0]$

5.7 Critical Resolved Shear Stress (τ_{crss}, yield stress)

- σ > yield stress => plastic deformation
- Slip is caused by shear stresses
  => resolved it on the **slip plane** and **slip direction**
  => resolved shear stress (τ_R).

$$\mathbf{A_n = A_{sp} \cos \theta}$$

A_n : cross section area perpendicular to the specimen axis

A_{sp} : the slip plane

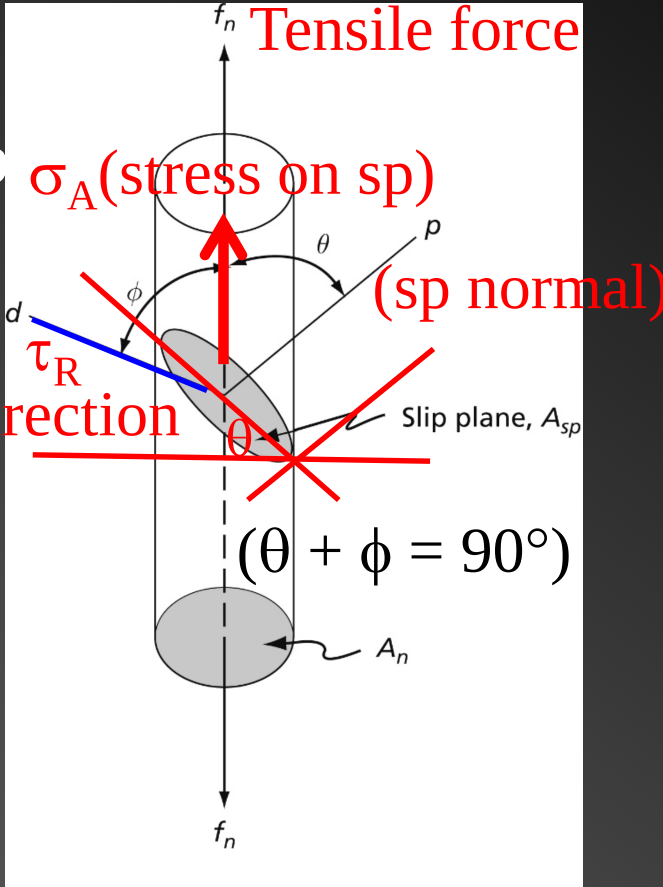


FIG. 5.14 A figure for the determination of the critical resolved shear stress equation

$$\tau_R = \frac{f_n \cos\phi}{A_{sp}} = (\sigma_A \cos\phi) = \frac{f_n \cos\phi}{\dfrac{A_n}{\cos\theta}} \qquad (A_n = A_{sp}\cos\theta)$$

$$= \frac{f_n}{A_n}\cos\theta\cos\phi = \sigma\cos\theta\cos\phi : \text{Schmid's law}$$

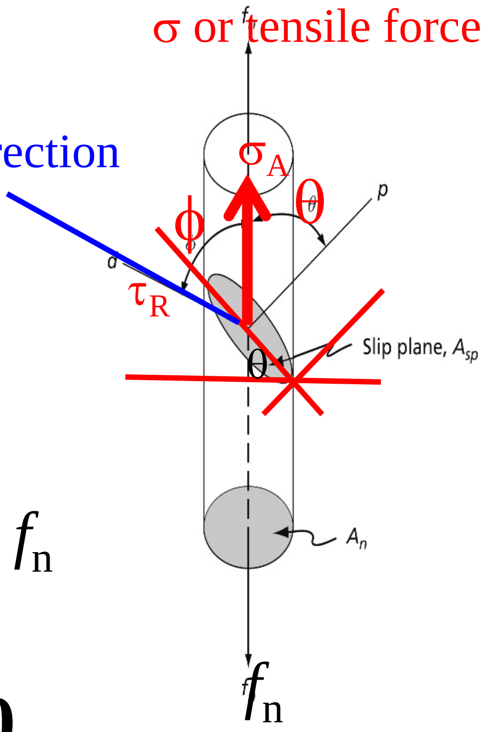


FIG. 5.14 A figure for the determination of the critical resolved shear stress equation

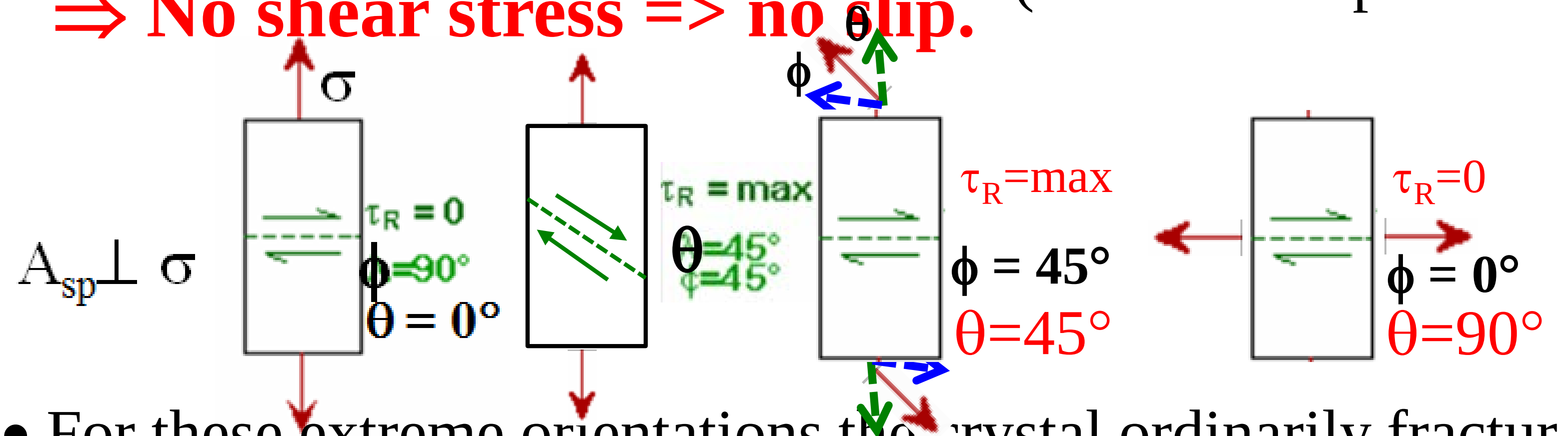
τ_R: resolved shear stress (**shear stress on the slip plane in the slip direction**)

σ_A : the stress on the slip plane (A_{sp}), in the direction of f_n

σ: f_n/A_n, the normal tensile stress on A_n

- Tensile axis ⊥ slip plane => $\phi = 90^o$ => $\boldsymbol{\tau_R = 0}$
- Tensile axis lies on the slip plane => $\theta = 90^o$ => $\boldsymbol{\tau_R = 0}$ (not in the slip direction)

⇒ No shear stress => no slip.



- For these extreme orientations the crystal ordinarily fractures rather than deforming plastically.

$$\tau_R = \sigma \cos\theta \cos\phi : \text{Schmid's law}$$

- Maximum shear stress (0.5σ) occurs when $\phi = \theta = 45^o$.

- In a given crystal, there may be **many available slip systems**.
- As the tensile load is increased, the resolved shear stress on each system increases until eventually $\boldsymbol{\tau}_{\mathbf{crss}}$ is reached on one system.

  $\Rightarrow$ The crystal begins to plastically deform by slip on this system, known as the **primary slip system**.

- if $\tau_R > \tau_{crss}$ **(critical resolved shear stress)**

  => plastic deformation by slip.

- The stress required to cause slip on the primary slip system is the yield stress of the single crystal.
- As the load is increased further, **τ_{crss}** may be reached **on other slip systems.**
- From Schmid's Law, it is apparent that the **primary slip system will be the system with the greatest Schmid factor (cos θ cos ϕ).**
- It is possible to calculate the values of **cos ϕ cos θ for every slip system and subsequently determine which slip system operates first.**

http://www.doitpoms.ac.uk/tlplib/slip/slip_geometry.php

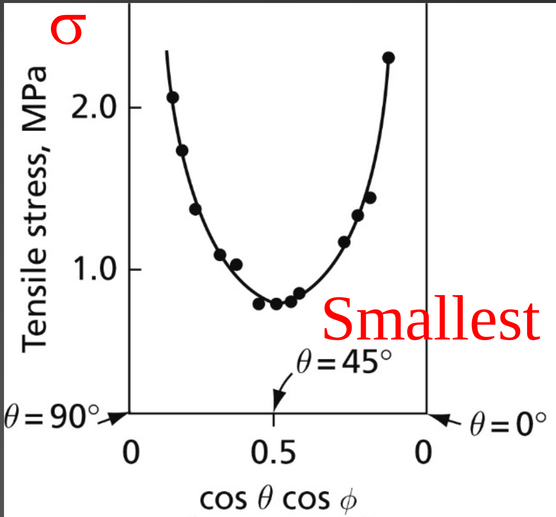


FIG. 5.15 The tensile yield point for magnesium single crystals of different orientations. Abscissae are values of the function cos *θ* cos *φ*. Smooth curve is for an assumed constant critical resolved shear stress of 63 psi (Burke, E. C., and Hibbard, W. R., Jr., *Trans. AIME*, **194**, 295 [1952].)

Smallest σ to get τ_{crss}

$$\tau_{crss}\ \textbf{(fixed)} = \sigma \cos\theta \cos\phi$$

τ_{crss}

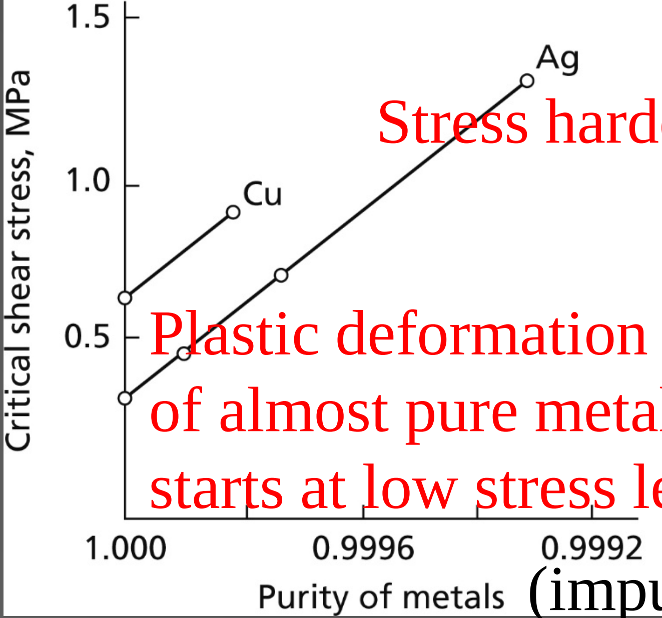


FIG. 5.16 Variation of the critical resolved shear stress with purity of the metal (After Rosi, F. D., *Trans. AIME*, **200**, 1009 [1954].)

Stress hardening

Plastic deformation of almost pure metals starts at low stress level.

(impurity ↑)

- τ is also a function of purity of the metal. The higher the purity, the lower the critical resolved shear stress (less stress hardening).

Temp. ↑ τ_{crss} ↓

temp. ↓ ⇒ critical shear stress ↑

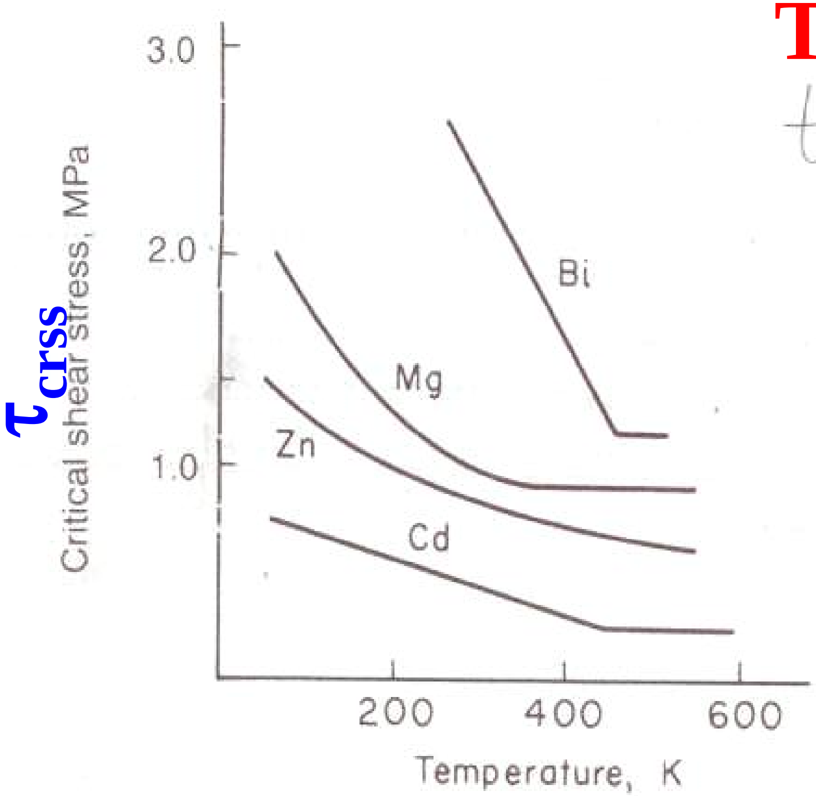


Fig. 5.17 Effect of temperature on the critical shear stress. *Note*: The data on which these curves are based predate those of Table 5.2. The higher critical stresses in this case correspond to crystals of lower purity. (Schmid, E., and Boas, W., *Kristallplastizität*, Julius Springer, Berlin, 1935.)

but very high

- **A higher τ_{crss}** implies a higher stress is necessary to plastically deform a metal, which in turn indicates the metal has **a high strength**.

⇒ FCC metals: many close-packed {111} planes, the **τ_{crss} is low** about 50 to 100 psi in a perfect crystal.

⇒ FCC metals have **high ductilities.**

⇒ BCC crystals: contain no close-packed planes, and we must exceed a higher τ_{crss}—on the order of 10,000 psi in perfect crystals—before slip occurs.

- BCC metals tend to have **high strengths and lower ductilities** compared to FCC metals.

The science and engineering of materials, p. 134

- **Strengthening of metallic materials:**

⇒ it is all about **preventing dislocation motion** and propagation (energetically unfavorable) for the dislocation to move or propagate.

⇒ For a material that has been **strengthened**, the amount of force required to start irreversible (plastic) deformation is greater than it was for the original material.

- There are five main strengthening mechanisms for metals

1. Work hardening

⇒ such as beating a red-hot piece of metal on anvil, has been used for centuries by blacksmiths to introduce dislocations into materials, increasing their **yield strengths**.

⇒ The primary species responsible for work hardening are dislocations.

⇒ **Dislocations interact with each other** by generating stress fields in the material, **impeding dislocation motion** by repulsive or attractive interactions.

2. Solid Solution Strengthening/Alloying (impurity)

- Solute atoms of one element are added to another, resulting in either **substitutional** or **interstitial** point defects in the crystal.
- The solute atoms **cause lattice distortions** that **impede dislocation motion**, increasing the yield stress of the material.

3. Precipitation Hardening (impurity)

- Alloying above a concentration given by the phase diagram will cause the formation of a second phase.
- The particles that compose the second phase precipitates act as **pinning points** in a similar manner to solutes.
- The dislocations in a material can **interact with the precipitate** atoms, which **block the motion** of dislocations within the crystal.

4. Grain Boundary (Grain Size) Strengthening

- In a polycrystalline metal, grain size has a tremendous influence on the mechanical properties.
- Grain boundaries act **as an impediment to dislocation motion** for the following two reasons:
  (1) Dislocation must change its direction of motion due to the differing orientation of grains.
  (2) Discontinuity of slip planes from one grain to another.

5. Transformation Hardening

- This method of hardening is used about **steels**.
- These steels use predominately higher levels of **C (2%)** and **Mn** along with heat treatment to increase strength.
- The steel must be heated to a temperature where the iron phase changes from ferrite into austenite, i.e. **changes crystal structure** from BCC (body centered cubic) to FCC (face centered cubic).

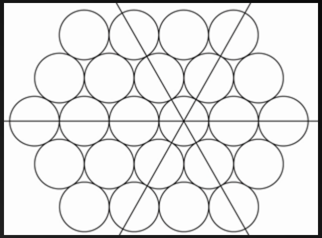
5.8 Slip On Equivalent Slip Systems

- For a crystal with equivalent slip systems, slip will start on the system with the **highest resolved shear stress (τ_R).**
- If several slip systems are equally stresses, slip occurs **simultaneously** at these systems.

5.9 The Dislocation Density

- Only small fraction of the dislocations formed during deformation come to the surface and are lost.
  => straining ↑ the **number of dislocations** in a metal increase
  => increase in **hardness** or **strength** of a metal.
- Dislocation density (ρ) :
  1. by estimating the length of the dislocation line (cm/cm^3)
  2. by the number of dislocation etch pits ($\#/cm^2$).

5.10 Slip Systems in Different Crystal Forms



Slip Systems in FCC:

- Close packed direction: <110>; four closed packed planes (octahedral plane): (111), (-111), (1-11), (11-1); Each octahedral plane have three slip directions => **4 × 3 = 12 slip systems.**
- These 12 slip systems are well distributed over space. Other systems have little chances to be activated.

- In general, plastically deformed FCC crystals slip on more than one octahedral planes.
  => a large number of **equivalent slip systems**.

- **Low critical resolved shear stress (τ_{crss})** for slip.

- **If the slip planes intersecting each other**, or mutual interference of dislocations gliding on intersecting slip planes, the stress required to produce additional deformation rises rapidly.

=> strain hardening (occur in **more than one slip system**)

(multiple glide)

Strain hardening and slip

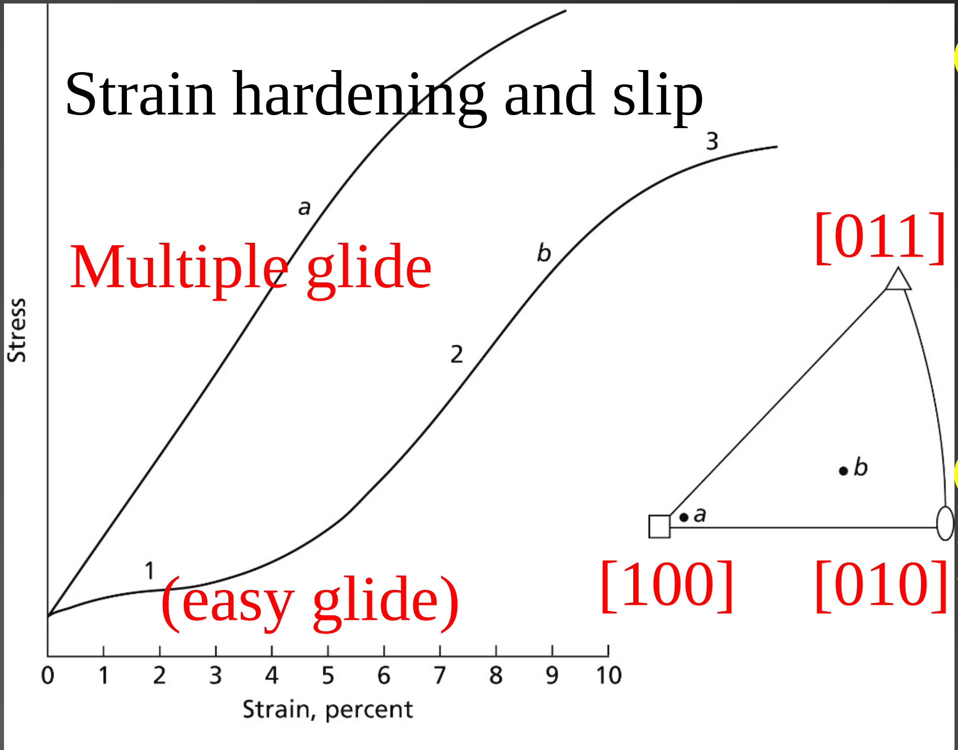


FIG. 5.17 Typical face-centered cubic single crystal stress-strain curves. Curve *a* corresponds to deformation by multiple glide from start of deformation; curve *b* corresponds to multiple glide after a period of single slip (easy glide). Crystal orientations are shown in the stereographic triangle

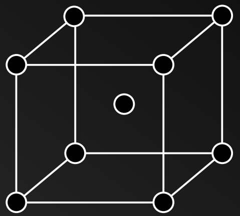
- The extend of the region of easy glide depends on specimen size and purity.
  => Large cross-section size and very high purity, easy glide tend to disappear.

Curve a: strain hardening close to <100>, several slip systems activated from beginning (**intersecting**) (τ_{crss} ↑) => steep slope (**multiple glide**)

Curve b:

1. slip on one plane (1) only (highly stressed (τ_{crss}↓), easy glide (2) & (3) are inactive. Strain hardening is minor (slip occurs on a single plane)
2. stage (2): much steeper slope, multiple glide on intersecting planes
   => strain hardening (ρ higher)
3. Rate of strain hardening decreases (ρ ↑ not much with γ)

Slip Systems in BCC:



- Close packed directions: <111>
- No truly closed-packed plane. The most closely packed plane (110); two close packed directions lie in this plane [-111] and [1-11].

  => **slip plane is not well defined**.

  ⇒ Any plane contains a close-packed <111> direction can act as a slip plane.
- further agreement: **higher** critical resolved shear stress (τ_{crss}) for slip, compared with that of FCC.

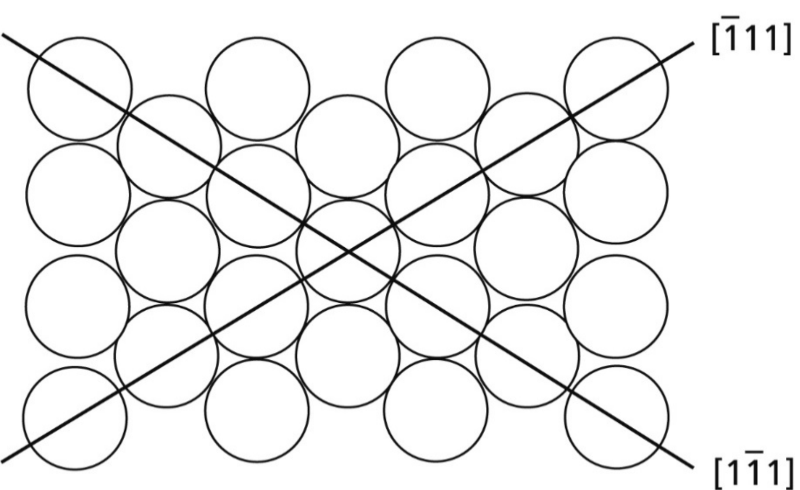


FIG. 5.19 The (110) plane of the body-centered cubic lattice

Slip Systems in HCP:

- (0001) basal plane is equivalent to octahedral planes of FCC; slip direction: <11-20>; This slip system has been observed in Zn, Cd, and Mg.
- **{10-10}(prism),<11-20>;** this slip system has been observed in Ti, Be (Be′ryllium), and Zr.
  => different τ_{crss}
- What is the reason for the deviations?
  => A complete solution is not available.

- If the atoms of hexagonal metals were truly spherical in shape, the ratio c/a would be the same **in all cases (1.632).**
  ⇒ However, different c/a values
  High c/a value => basal plane dominates
  Low c/a value => other slip plane is packed more densely

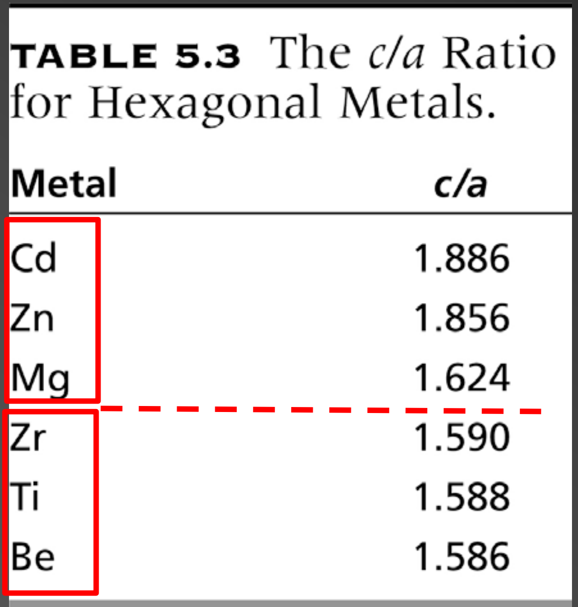


TABLE 5.3 The *c/a* Ratio for Hexagonal Metals.

Metal	*c/a*
Cd	1.886
Zn	1.856
Mg	1.624
Zr	1.590
Ti	1.588
Be	1.586

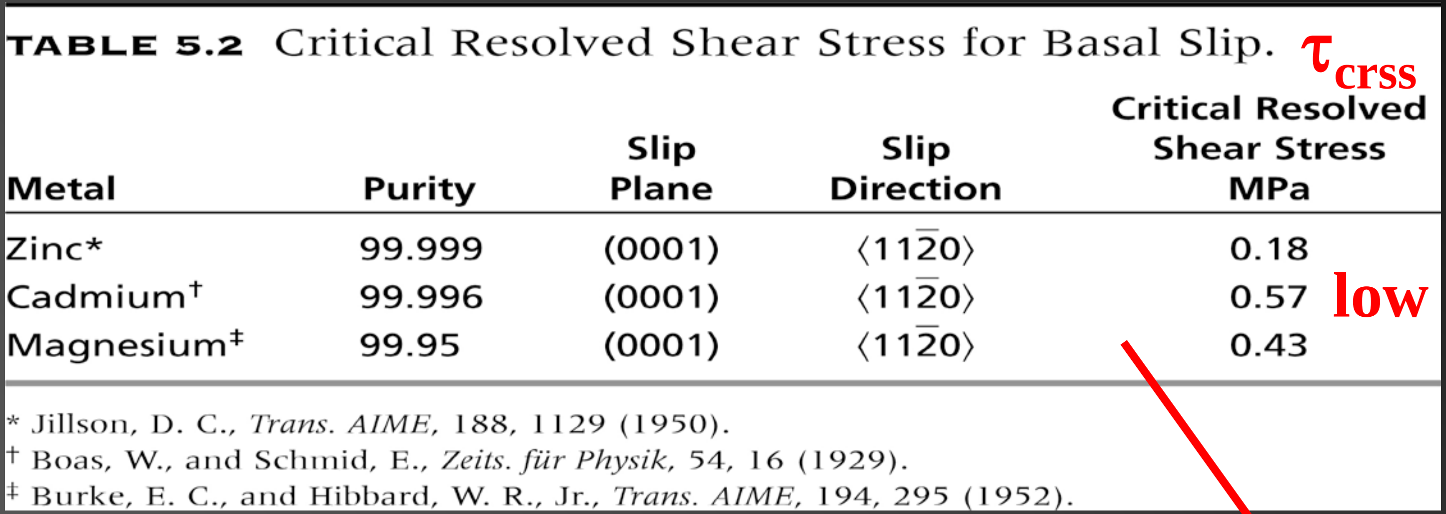


TABLE 5.2 Critical Resolved Shear Stress for Basal Slip. τ_{crss}

Metal	Purity	Slip Plane	Slip Direction	Critical Resolved Shear Stress MPa
Zinc*	99.999	(0001)	$\langle 11\bar{2}0 \rangle$	0.18
Cadmium†	99.996	(0001)	$\langle 11\bar{2}0 \rangle$	0.57
Magnesium‡	99.95	(0001)	$\langle 11\bar{2}0 \rangle$	0.43

low

---

* Jillson, D. C., *Trans. AIME*, 188, 1129 (1950).

† Boas, W., and Schmid, E., *Zeits. für Physik*, 54, 16 (1929).

‡ Burke, E. C., and Hibbard, W. R., Jr., *Trans. AIME*, 194, 295 (1952).

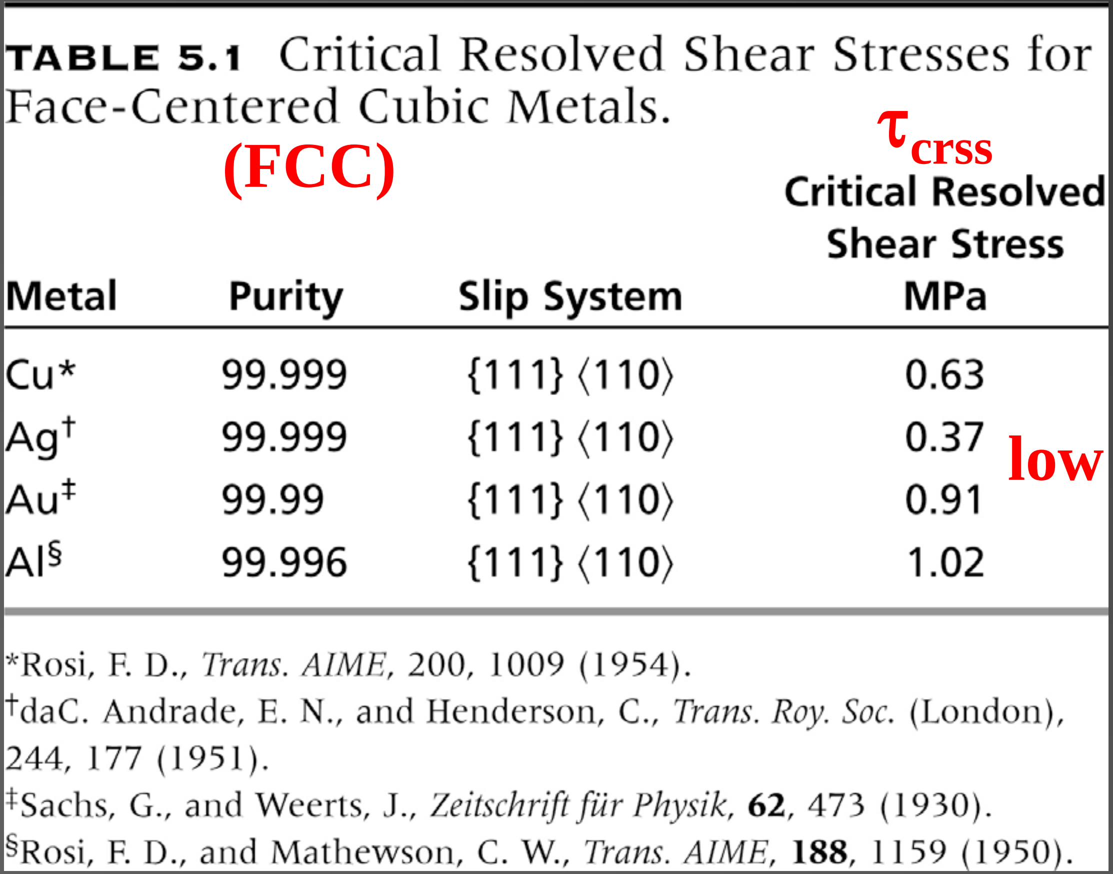


TABLE 5.1 Critical Resolved Shear Stresses for Face-Centered Cubic Metals.

(FCC)

τ_{crss}

Metal	Purity	Slip System	Critical Resolved Shear Stress MPa
Cu*	99.999	{111} ⟨110⟩	0.63
Ag†	99.999	{111} ⟨110⟩	0.37
Au‡	99.99	{111} ⟨110⟩	0.91
Al§	99.996	{111} ⟨110⟩	1.02

low

---

*Rosi, F. D., *Trans. AIME*, 200, 1009 (1954).

†daC. Andrade, E. N., and Henderson, C., *Trans. Roy. Soc.* (London), 244, 177 (1951).

‡Sachs, G., and Weerts, J., *Zeitschrift für Physik*, **62**, 473 (1930).

§Rosi, F. D., and Mathewson, C. W., *Trans. AIME*, **188**, 1159 (1950).

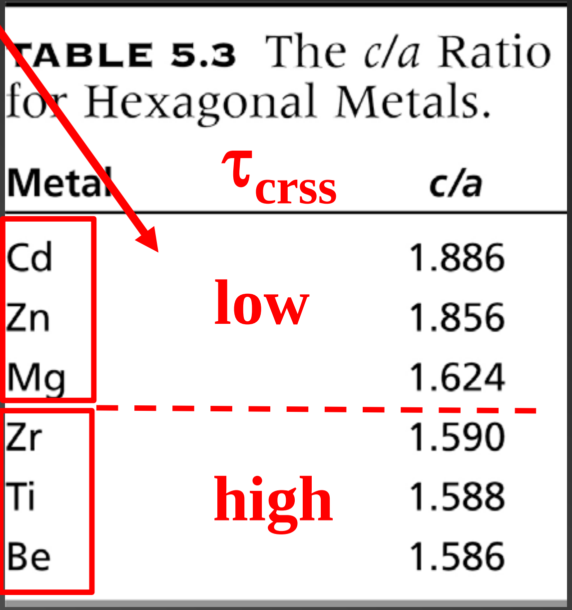


TABLE 5.3 The *c/a* Ratio for Hexagonal Metals.

Metal	τ_{crss}	*c/a*
Cd	low	1.886
Zn		1.856
Mg		1.624
Zr	high	1.590
Ti		1.588
Be		1.586

- HCP metals: contain close-packed basal planes, to have **low τ_{crss}**.

⇒ Zn: a c/a ratio greater than or equal to the theoretical ratio of 1.632
τ_{crss} is less than 100 psi, just as in FCC metals.

⇒ **No cross slip.**

⇒ **Brittle.**

⇒ Ti: however, the c/a ratio is less than 1.632
the close-packed planes are spaced too closely together
Slip now occurs on planes such as , **the "prism" planes**
τ_{crss} is then as **great** as or greater than in BCC metals.

⇒ **Cross slip**

⇒ **Less brittle (ductility)**

The science and engineering of materials, p. 134

Ductility: FCC > BCC > HCP > cubic

(Au)

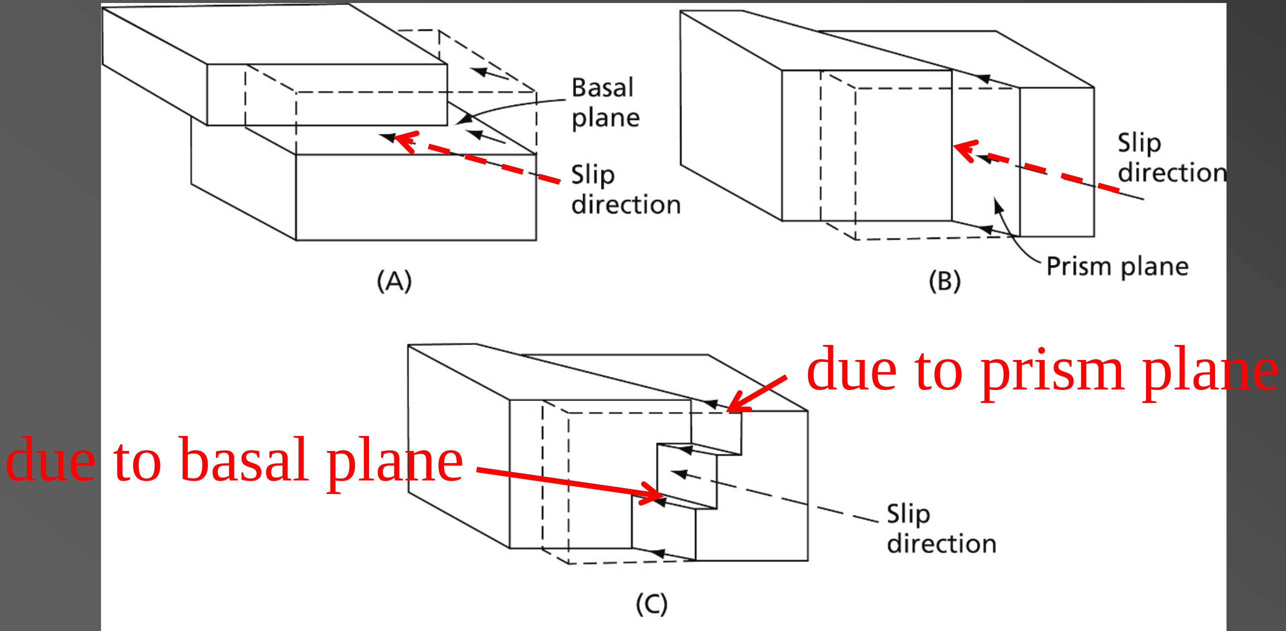
↓

(small c/a > big c/a)

The science and engineering of materials, p. 134

5.11 Cross-Slip (**planes**, different from dislocation intersection)

- A phenomenon that occurs in crystal when there are two or more slip planes with a common slip direction.
- e.g. Mg, dislocation could move from prism plane {10-10} to basal plane {0001}.
  => slip direction <11-20>
  => part in basal and part in prism planes: staircase.



due to prism plane

due to basal plane

FIG. 5.20 Schematic representation of cross-slip in a hexagonal metal: **(A)** Slip on basal plane, **(B)** slip on prism plane, and **(C)** cross-slip on basal and prism planes

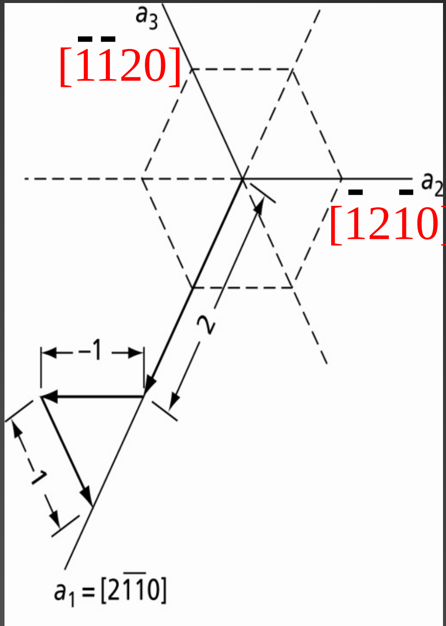
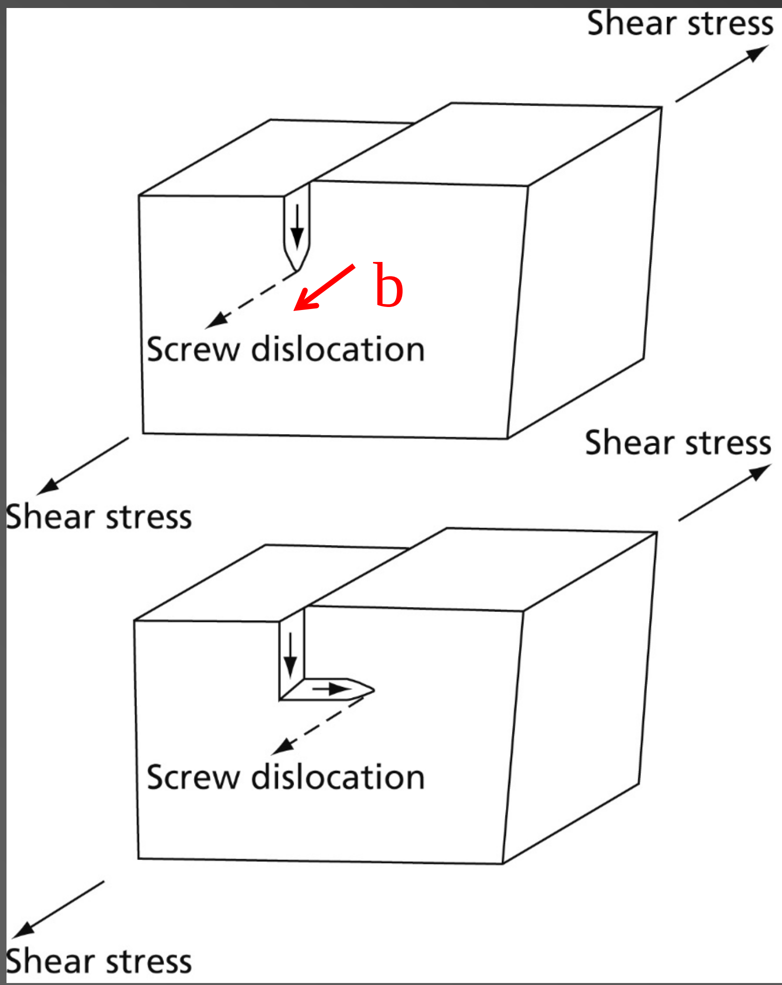


FIG. 1.18 Determination of indices of a digonal axis of Type I—$[2\bar{1}\bar{1}0]$

- Can only occur for a screw dislocation (b//dislocation line).
- Edge dislocations confined to move in a single slip plane.



b: Burgers vector

- **Screw dislocations move in different planes**

FIG. 5.22 Motion of a screw dislocation during cross-slip. In the upper figure the dislocation is moving in a vertical plane, while in the lower figure it has shifted its slip plane so that it moves horizontally

• Obstacles by-pass can be made by cross slip

Prism plane {10-10}

basal plane
(0001)

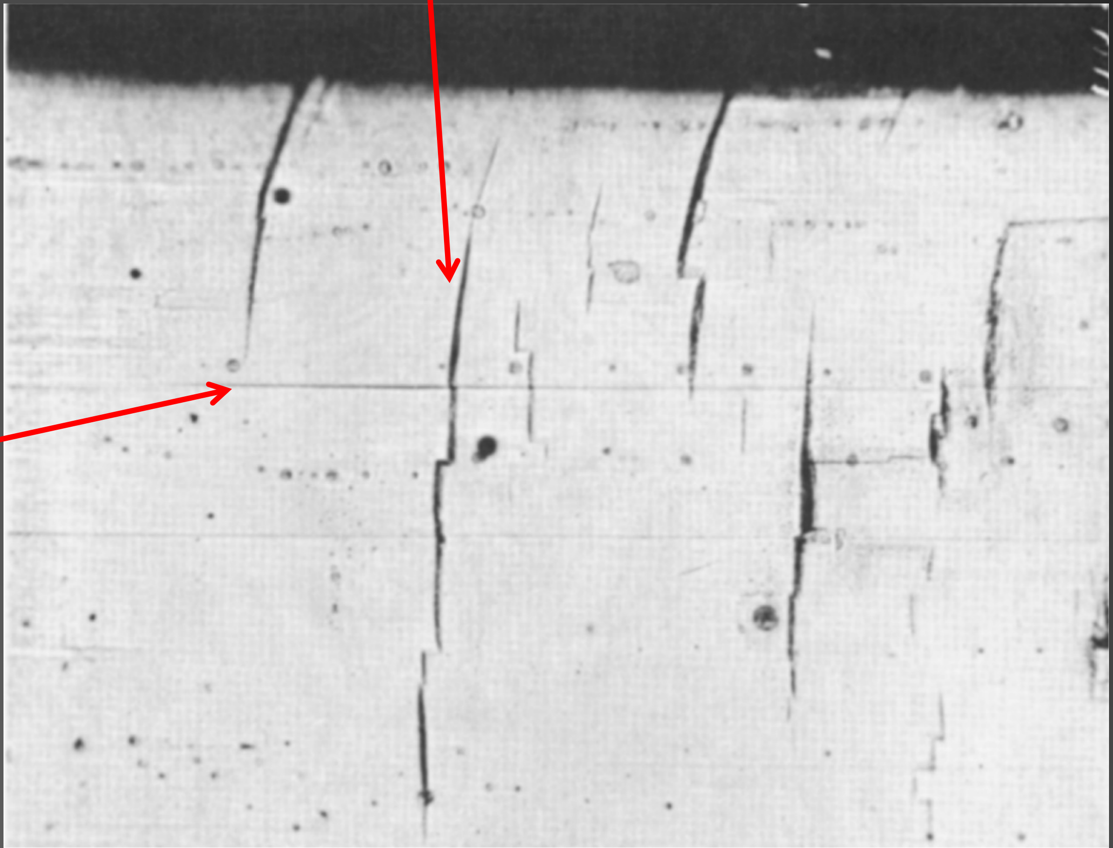


FIG. 5.21 Cross-slip in magnesium. The vertical slip plane traces correspond to the {1010} prism plane, whereas the horizontal slip plane traces correspond to the basal plane (0002). 290 × (Reed-Hill, R. E., and Robertson, W. D., *Trans. AIME*, **209** 496 [1957].)

- In many HCP metals, no cross-slip can occur because the slip planes are parallel (**not intersecting slip systems**).

⇒Zn: a c/a ratio greater than or equal to the theoretical ratio of 1.632

⇒**No cross slip (brittle).**

c/a ~ 1.624

- **A single crystal of Mg (HCP)** can be stretched into a ribbon-like shape **four ~ six times** its original length because slip in HCP metals will occur readily on the basal plane—the primary slip plane. When **a single crystal** is deformed, assuming the **basal plane** is suitably oriented with applied stress, **a very large deformation can occur**.

The science and engineering of materials, p. 135

- However, **polycrystalline Mg** shows limited ductilities (brittle).

⇒ **Each crystal must deform** such that the strain developed in any one crystal is accommodated by its neighbors. Since there is **no intersecting slip systems**, dislocations **cannot glide** from one slip plane in one crystal (grain) onto another slip plane in a neighboring crystal.

⇒ As a result, polycrystalline HCP metals (such as Mg) show limited ductility (brittle).

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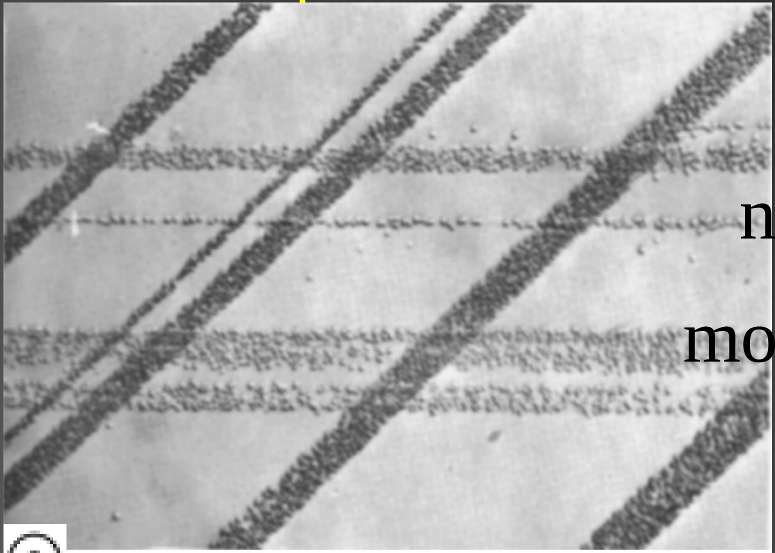
5.12 Slip Bands:

- A group of closely spaced slip (dislocation) lines that appear, at low magnification, to be a single large line.
- Wavy and irregular
  => Because the dislocations that produce the bands are **not so confined as to move in a single plane.**
- The shifting of the dislocations from one slip plane to another is usually the result of the **cross-slip of screw dislocations**.
- Require the use of an electron microscope to see the individual line.



Etch pits

narrow

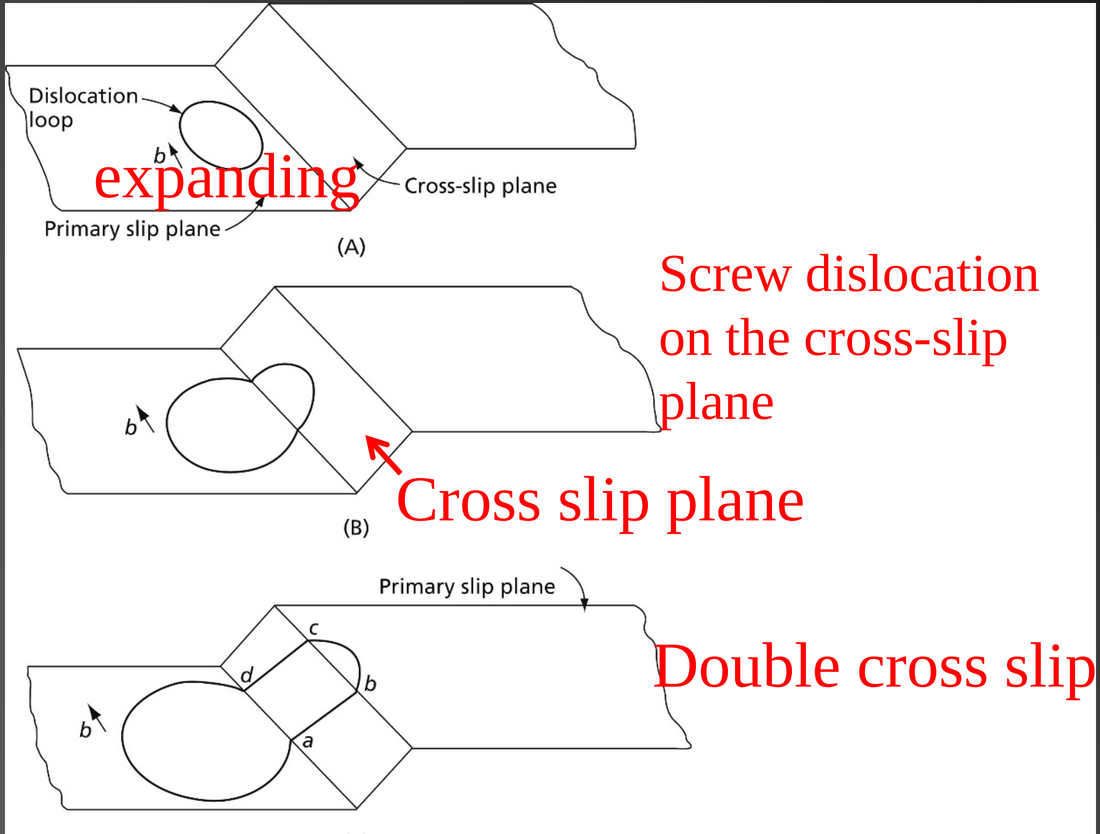
moderate

FIG. 5.24 Slip bands in LiF. Bands formed at −196 °C and 0.36 percent strain (Reprinted with permission from J.J. Gilman and W.G. Johnson, *Journal of Applied Physics*, Vol. 30, Issue 2, Page 129, Copyright 1959, American Institute of Physics)

5.13 Double Cross-Slip:

- Double cross slip: the dislocation multiplication (moving).
- The nucleated dislocations
  => First forms narrow slip bands
  => With continued straining, the slip bands expand effectively.

- bc: dislocations can be created on the new slip plane.
- Similar to Frank-Read source.
- These freshly created dislocations will not have enough time to become pinned by impurity atoms.
- More probable than grown-in dislocations.



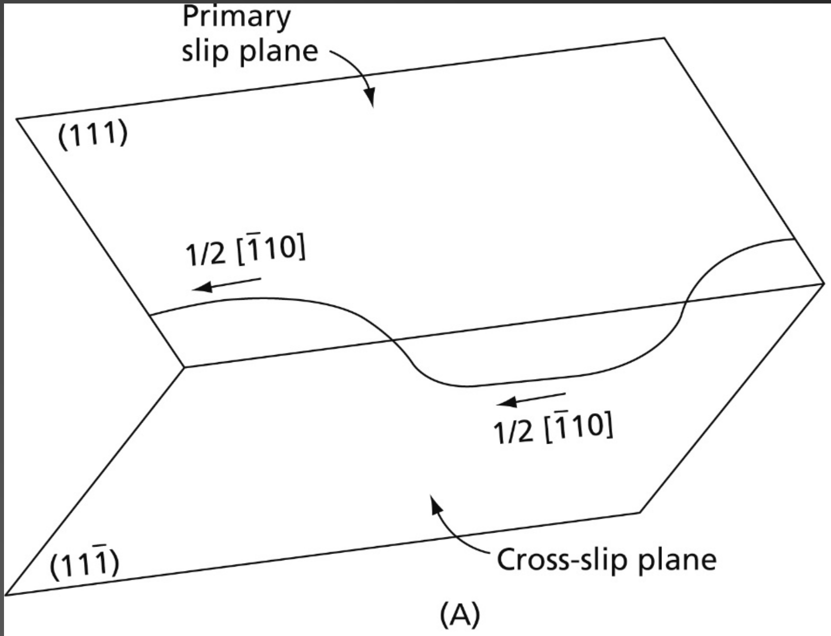
expanding

Screw dislocation on the cross-slip plane

Cross slip plane

Double cross slip

FIG. 5.25 Double cross-slip



5.14 Extended Dislocations and Cross-Slip

Common slip direction [-110]

FCC: b = c + d **=>**

$$\frac{1}{2}[\bar{1}10]=\frac{1}{6}[\bar{1}2\bar{1}]+\frac{1}{6}[\bar{2}11]$$

- For a dislocation to lie on the slip plane, the inner product of Burgers vector and surface normal has to be zero.

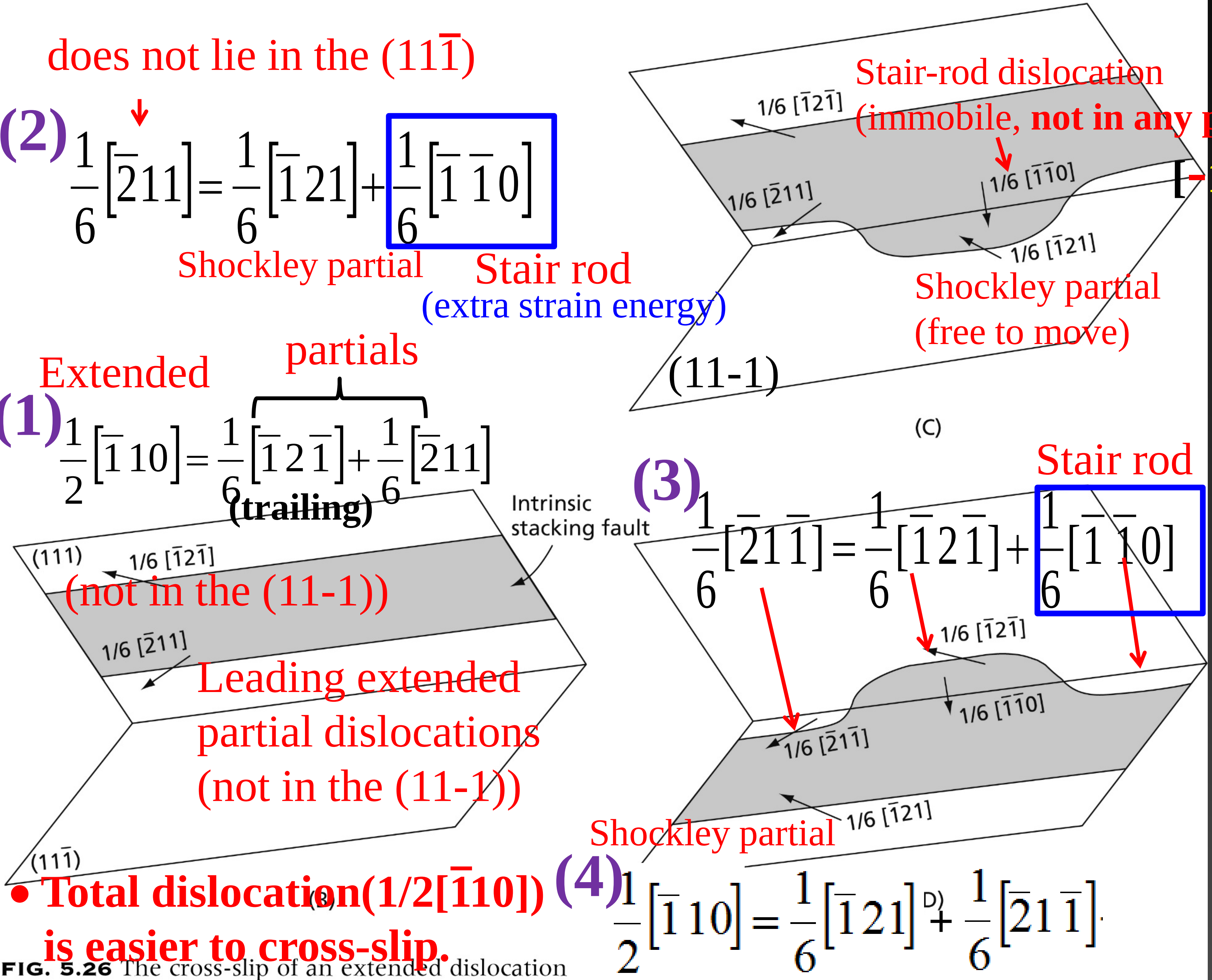
- ½[-110] can readily cross-slip between (111) and (11$\bar{1}$)

(total **(extended)**

$$\frac{1}{2}[\bar{1}10]=\frac{1}{6}[\bar{1}2\bar{1}]+\frac{1}{6}[\bar{2}11]$$

Different planes

FIG. 5.26 The cross-slip of an extended dislocation



does not lie in the (111̄)

(2)
$$\frac{1}{6}[\bar{2}11]=\frac{1}{6}[\bar{1}21]+\boxed{\frac{1}{6}[\bar{1}\,\bar{1}0]}$$

Shockley partial

Stair rod

(extra strain energy)

Stair-rod dislocation (immobile, **not in any plane**)

[-1-10]

Shockley partial (free to move)

(11-1)

Extended

(1)
$$\frac{1}{2}[\bar{1}10]=\overbrace{\underbrace{\frac{1}{6}[\bar{1}2\bar{1}]}_{\text{(trailing)}}+\frac{1}{6}[\bar{2}11]}^{\text{partials}}$$

(not in the (11-1))

Leading extended partial dislocations (not in the (11-1))

(3)
$$\frac{1}{6}[\bar{2}1\bar{1}]=\frac{1}{6}[\bar{1}2\bar{1}]+\boxed{\frac{1}{6}[\bar{1}\,\bar{1}0]}$$

Stair rod

Shockley partial

(4)
$$\frac{1}{2}[\bar{1}10]=\frac{1}{6}[\bar{1}21]+\frac{1}{6}[\bar{2}1\bar{1}]$$

- **Total dislocation(1/2[1̄10]) is easier to cross-slip.**

FIG. 5.26 The cross-slip of an extended dislocation leading $\boxed{(111)}$ trailing

$$\begin{tikzcd}
\frac{1}{2}[\bar{1}\,1\,0] = \arrow[dd] & \frac{1}{6}[\bar{2}\,1\,1] \arrow[d] & + & \frac{1}{6}[\bar{1}\,2\,\bar{1}] \\
 & \boxed{\frac{1}{6}[\bar{1}\,\bar{1}\,0]} & & + \ \boxed{\frac{1}{6}[\bar{1}\,\bar{1}\,0]} \arrow[d] \\
\frac{1}{2}[\bar{1}\,1\,0] = & + \ \frac{1}{6}[\bar{1}\,2\,1] & + & \frac{1}{6}[\bar{2}\,1\,\bar{1}]
\end{tikzcd}$$

6/36

6/36

$\cancel{(11\bar{1})}$

2/36

Stair rod

6/36

6/36

$\boxed{(11\bar{1})}$

5.15 Crystal Structure Rotation during Tensile and Compressive Deformation

- When a single crystal is deformed in either tensile or compression => rotation for crystal lattice
- In tensile
  => align **the slip plane and the active slip direction** parallel to the tensile stress axis.

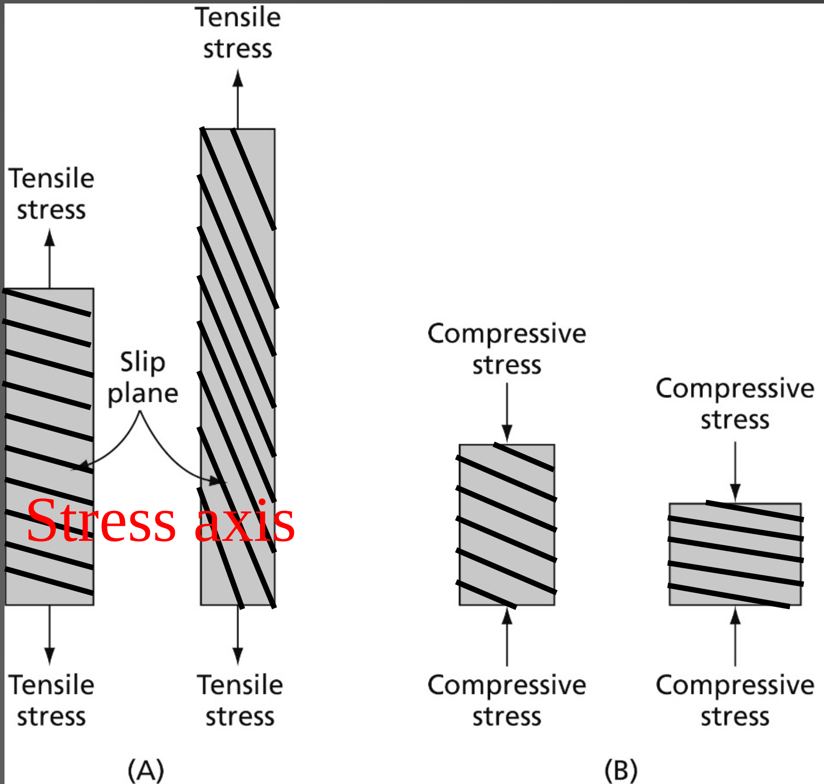
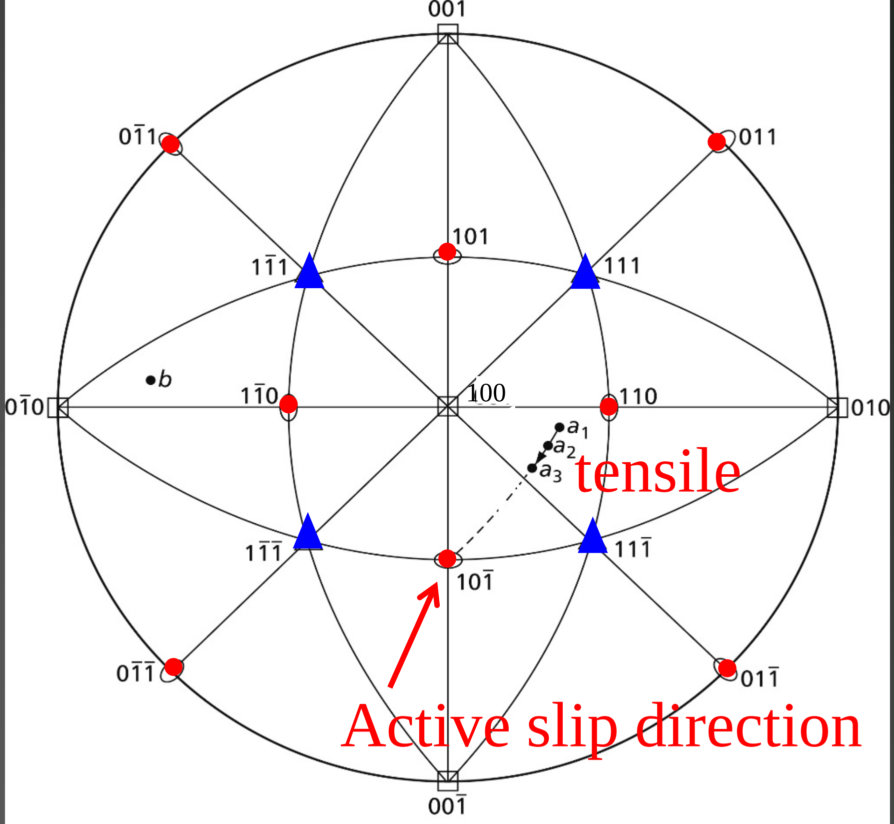


FIG. 5.27 Rotation of the crystal lattice in tension and compression

- **In compression: slip plane ⊥ stress axis (slip plane normal // stress axis)**



tensile

Active slip direction

FIG. 5.28 In tension the lattice rotation is equivalent to a rotation of the stress axis (a) toward the slip direction. This stereographic projection shows this rotation in a face-centered cubic crystal

Stress axis $\mathbf{a_1}$

[111]
or
$[1\bar{1}1]$

d: Slip direction

$[10\bar{1}]$

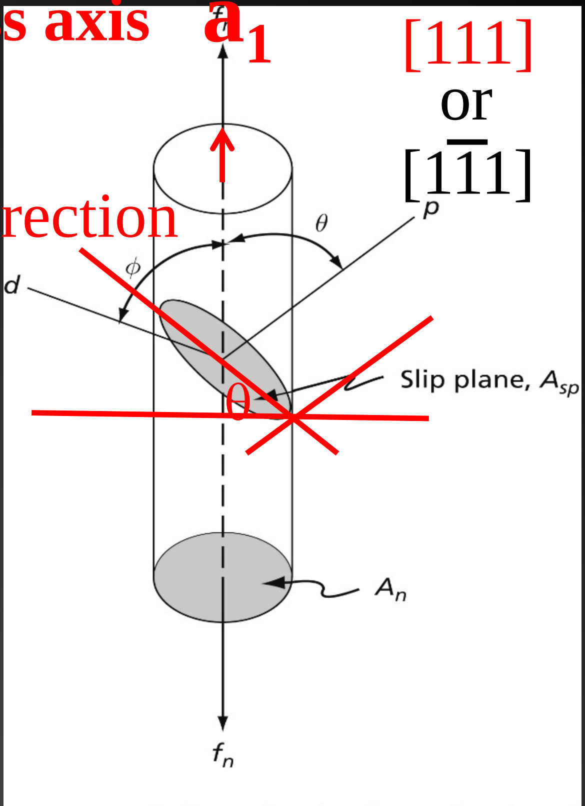
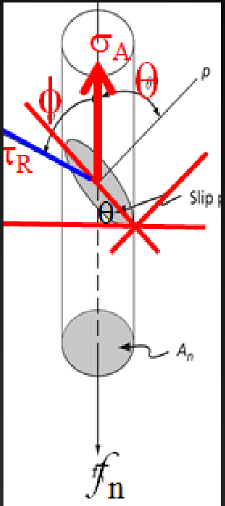


FIG. 5.14 A figure for the determination of the critical resolved shear stress equation

- An FCC crystal: initial orientation (**slip plane) at $\mathbf{a_1}$**, after **tension** the Laue pattern shows the direction move to a_2, and repeat the process => a_3 => connecting the a_1-a_2-a_3 should go through the **active slip direction**.
- **A general rule** applicable to the tensile deformation of FCC => The closest <110> direction to the stress axis that can be reached by crossing a boundary of the stereographic triangle that contains the stress axis.

- [10$\bar{1}$] lies in both (111) and (1$\bar{1}$1) planes.
- Slip plane orientation: a_1 ([43$\bar{1}$])
  => a_1 & [111]: ~ 47°, a_1 & [1$\bar{1}$1]: ~ 90°
  => (111) is the active slip plane
  => higher resolved shear stress



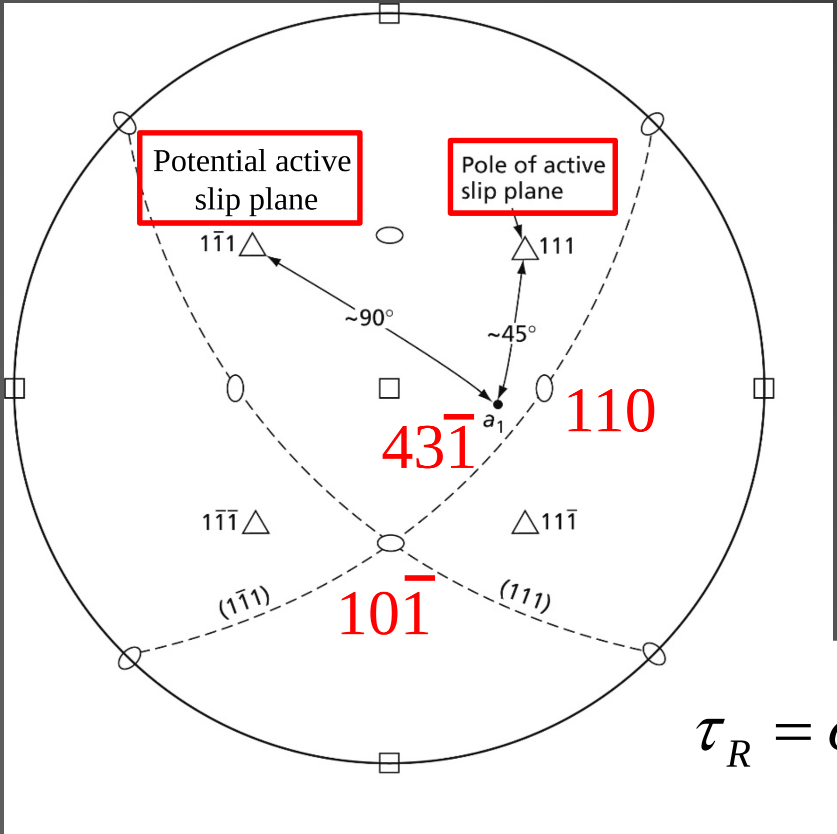


FIG. 5.29 The original stress axis orientation in Fig. 5.28 lies about 45° from the pole of the (111) plane and about 90° from the pole of (1$\bar{1}$1). These are the two slip planes that contain the active slip direction

θ

[43-1] • [111] ⇒ θ ~ 47°

[43-1] • [1-11] ⇒ θ ~ 90°

Primary slip system: (111) [10$\bar{1}$]

Cross slip system: (1$\bar{1}$1) [10$\bar{1}$]

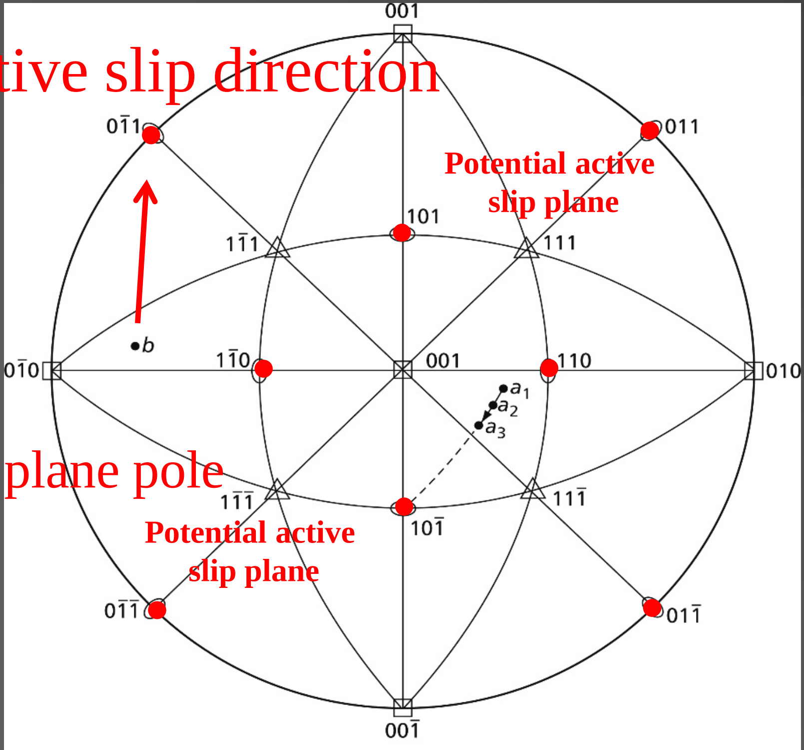
$$\tau_R = \sigma_A \cos\phi = \frac{f_n}{A_n}\cos\theta\cos\phi = \sigma\cos\theta\cos\phi$$

- Stress axis: b $[3\bar{5}1]$
- Active slip direction : $[0\bar{1}1]$.
  => $[1\bar{1}\bar{1}]$ is the active slip plane
  => higher resolved shear stress
- The indicated slip directions and slip planes yield the active slip systems, **highest resolved shear stress**.

b [3-51]

b • [1-1-1]; θ ~ 47°

b • [111]; θ ~ 96°



Active slip direction

Potential active slip plane

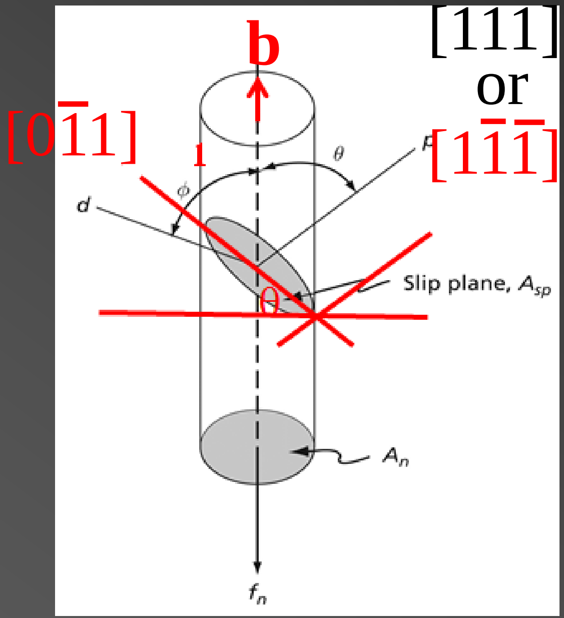
Slip plane pole

Potential active slip plane

FIG. 5.28 In tension the lattice rotation is equivalent to a rotation of the stress axis (*a*) toward the slip direction. This stereographic projection shows this rotation in a face-centered cubic crystal

Primary slip system: $(1\bar{1}\bar{1})$ $[0\bar{1}1]$

Cross slip system: (111) $[0\bar{1}1]$



- In compression:
- Active slip direction : [10$\bar{1}$].

=> a moves toward (111)

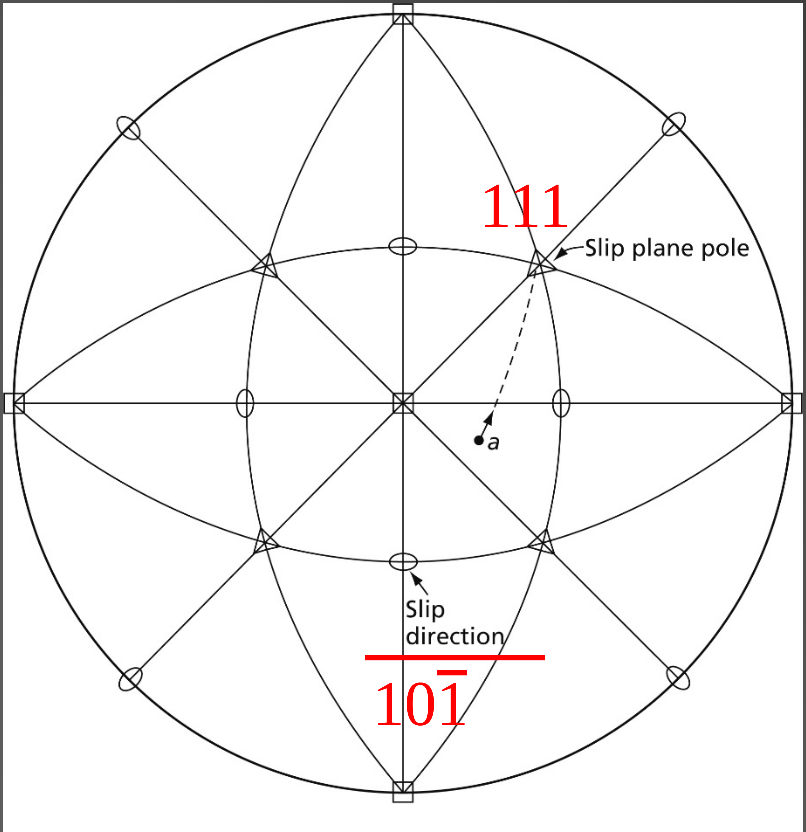


FIG. 5.30 In compression, the stress axis (a) rotates toward the pole of the active slip plane

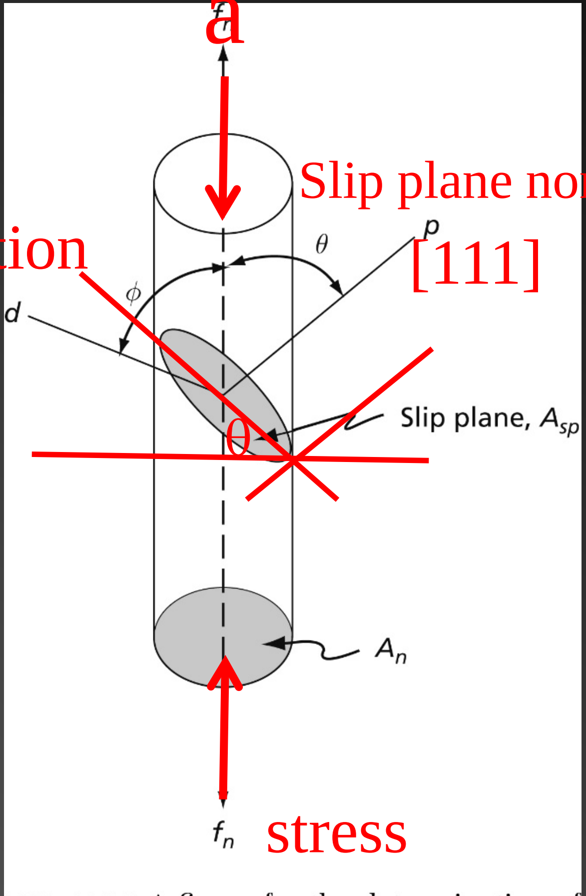


FIG. 5.14 A figure for the determination of the critical resolved shear stress equation

Primary slip system: (111) [10$\bar{1}$]

Cross slip system: (1$\bar{1}$1) [10$\bar{1}$]

5.16 The Notation For The Slip Systems in The Deformation of FCC Crystals

- The slip system with the highest resolved shear stress
  => **primary slip system**; $(111)[10\bar{1}]$.
- If the dislocation of this system were to cross-slip
  => move to another plane which contains $[10\bar{1}]$ slip direction
  => $(1\bar{1}1)$ $[10\bar{1}]$ (**cross-slip system** for the stress axis orientation a_1).

Primary slip system: (111) $[10\bar{1}]$

Cross slip system: $(1\bar{1}1)$ $[10\bar{1}]$

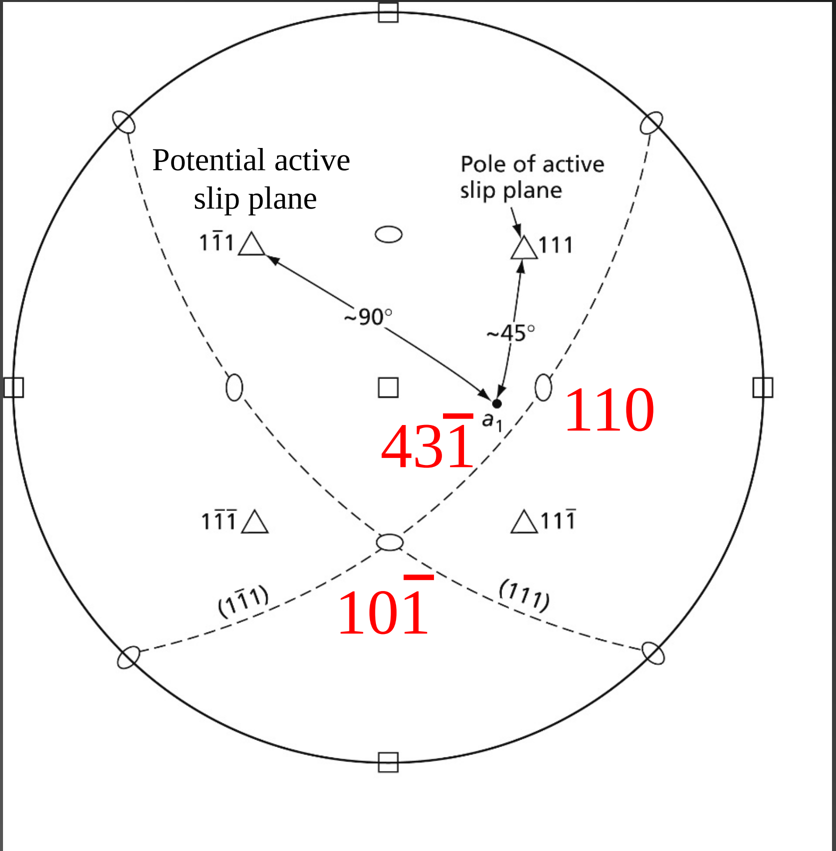
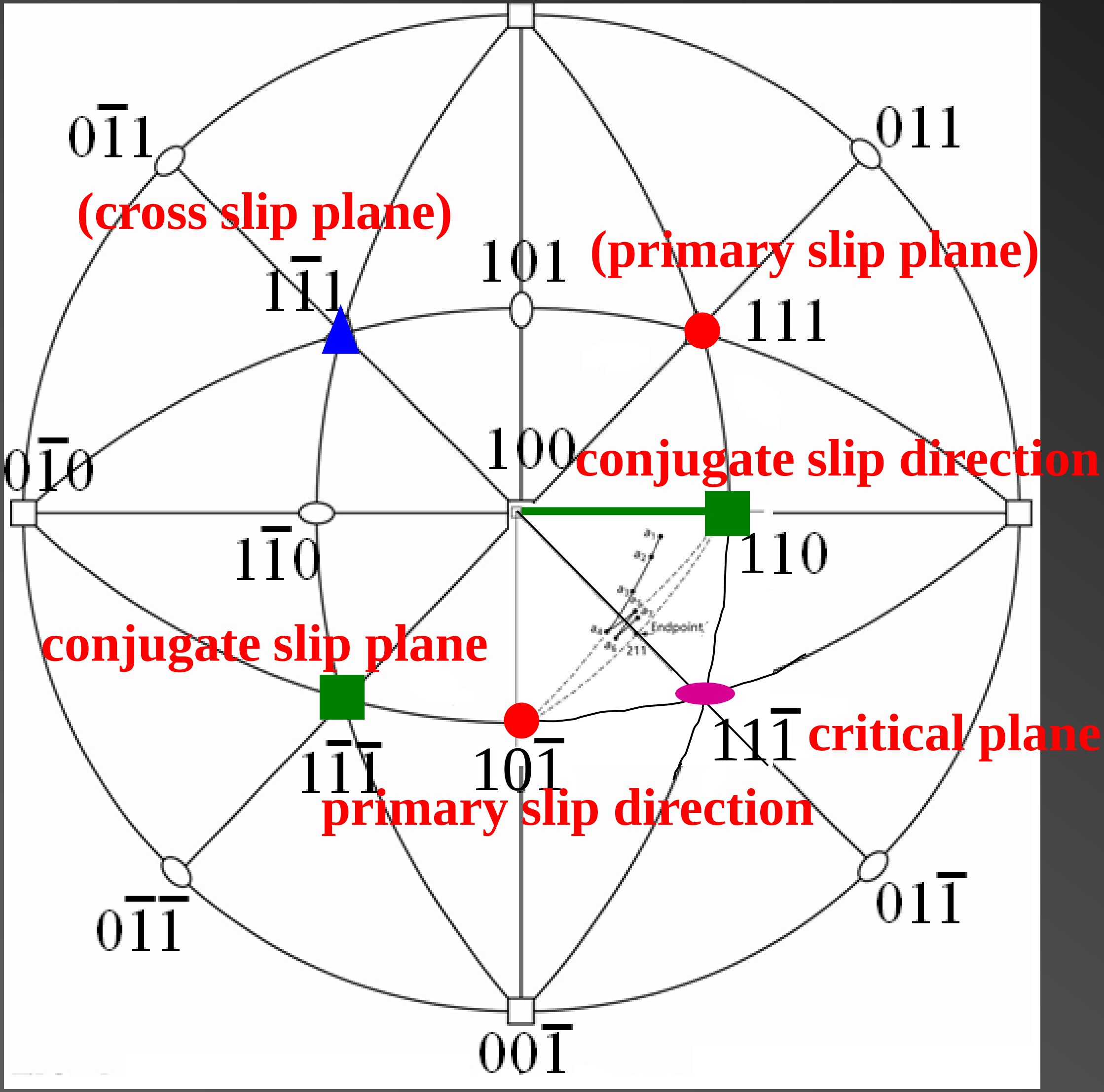
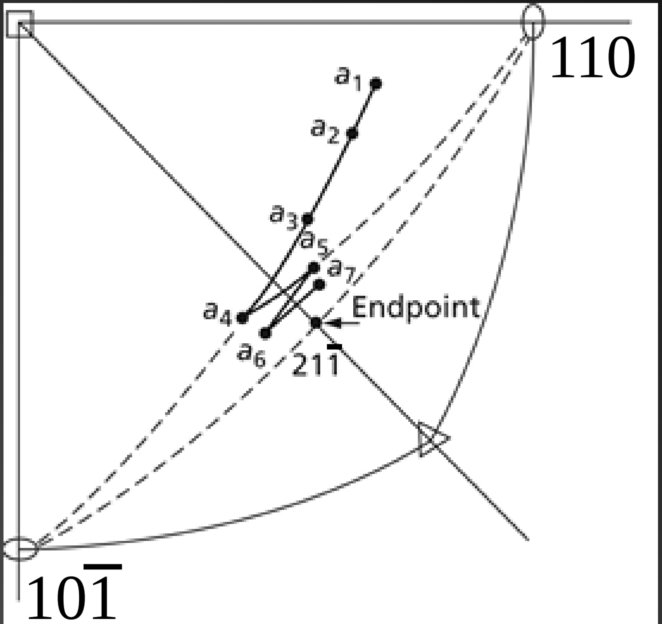


FIG. 5.29 The original stress axis orientation in Fig. 5.28 lies about 45° from the pole of the (111) plane and about 90° from the pole of (111). These are the two slip planes that contain the active slip direction





Critical plane: Rosi and Mathewson saw slip lines on it only when the crystals had a critical orientation: their tensile axes lay on the green boundary. (Recrystallization, grain growth, and textures p. 4)

F. D. Rosi and C. H. Mathewson, Trans AIME, 188, 1159 (1950)

- **Conjugate** slip system: happen once the rotation of the crystal out of its original stereographic triangle into the one adjoining it.
  => resolved shear stress is greater on the $(1\bar{1}\bar{1})[110]$ slip system.
  $\Rightarrow a_3 \rightarrow a_4$
- The crystal will continue to rotate with deformation occurring on alternating slip.
- The **overshoot** between the primary slip system and conjugate slip system will eventually settled at $[21\bar{1}]$ direction – a direction that lies on the same great circle as the conjugate and primary slip directions and midway between them.
  => **two slip systems are equally favored (double slip),** stable end
  ⇒ Strain hardening (multiple glide)
  ⇒ further deformation will not change the orientation of the crystal relative to the tensile stress axis.

⇒ The crystal will neck down until fracture without changing orientation.

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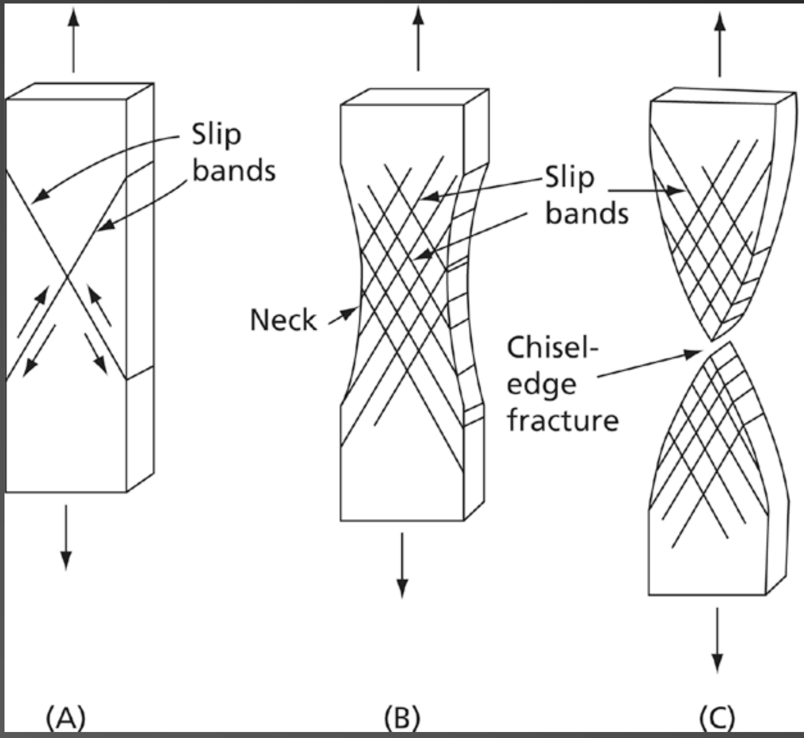


FIG. 21.3 (A) Crystal oriented for double slip. **(B)** Development of a neck. **(C)** Chisel edge fracture

- This is one reason why the ductility in **cubic** is much less compared to HCP, which deforms by **single slip**.

Ductility: FCC > BCC > HCP > cubic

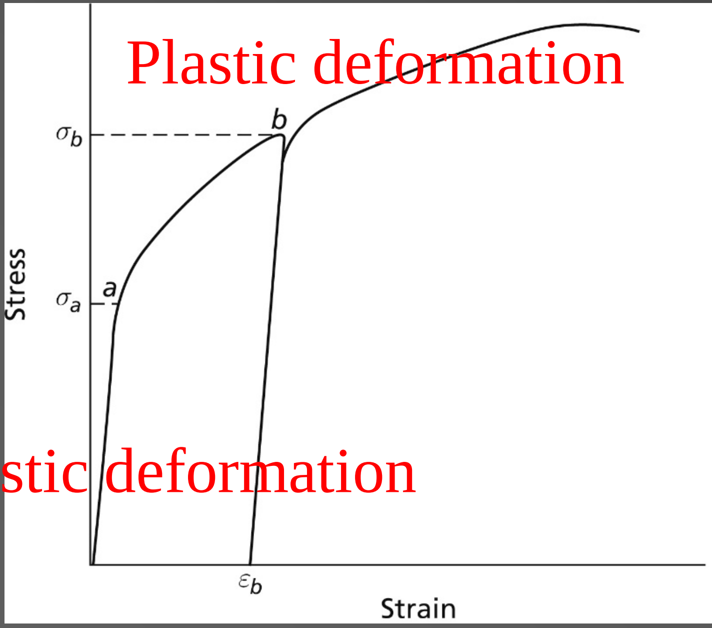
(Au) (small c/a > big c/a)

DISLOCATIONS AND MECHANICAL BEHAVIOUR OF MATERIALS p. 144

$\Rightarrow$ FCC metals: many close-packed {111} planes, the τ_{crss} is low

5.17 Work Hardening

- When a metal is deform to a strain ε_b and unloaded, it will not begin to deform until the stress reaches σ_b.
  => the flow stress is raised from σ_a to σ_b, due to previous deformation (assume recovery rate is low)
  => **work hardening**



Plastic deformation

Elastic deformation

FIG. 5.32 Normally when a metal is deformed to a strain such as ϵ_b and then it is unloaded, it will not begin to deform until the stress is raised back to σ_b. The strain ϵ_b raises the flow stresses from σ_a to σ_b.

- Engineering stress and strain: expressed in terms of **original** sample dimension (A_0, l_0).
  $\sigma_e = P/A_0$ (P: load); $\varepsilon_e = (l_i - l_0)/l_0 = \Delta l / l_0$
- This does not take into account the reduction in area at the neck.

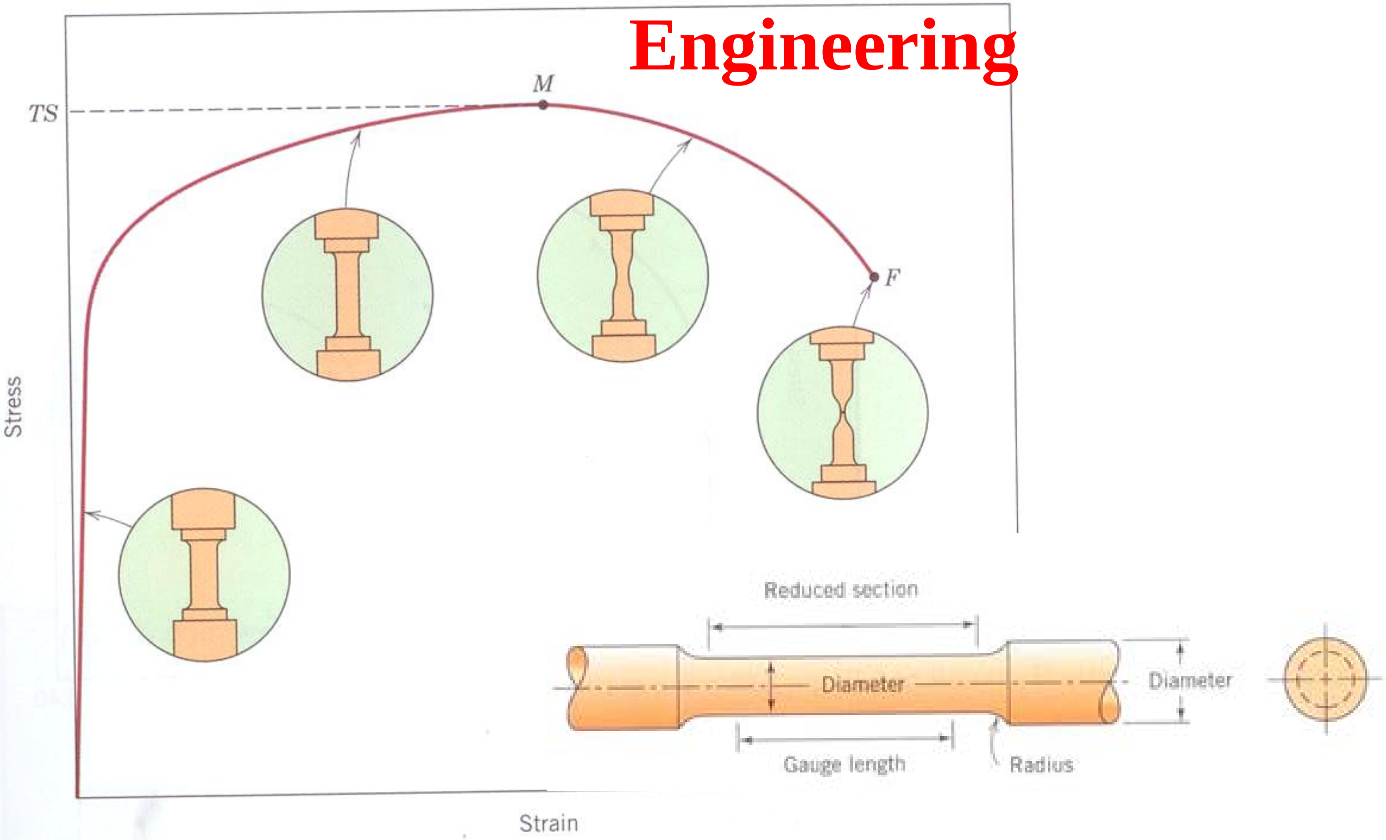
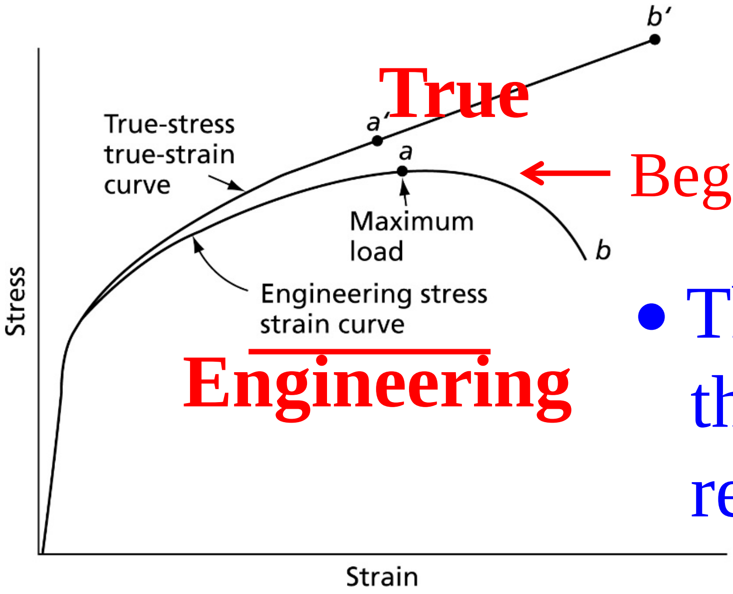


Figure 7.11 Typical engineering stress–strain behavior to fracture, point *F*. The tensile strength *TS* is indicated at point *M*. The circular insets represent the geometry of the deformed specimen at various points along the curve.

Figure 7.2 A standard tensile specimen with circular cross section.

- To better illustrate the nature of the work hardening
  => use **true stress and strain** instead of engineering stress and strain.
- True stress and strain: expresses in terms of **instantaneous** cross-section area (A_i) and gauge length (l_i).

$$\sigma_t = P/A_i \text{ (P: load); } \varepsilon_t = \Delta l/l_i \, (\Delta l = l_i - l_0)$$



← Begin to neck

FIG. 5.33 A comparison between an engineering stress-strain curve and the corresponding true-stress and true-strain curve.

- The corrected a → a' takes into account the complex stress state within the neck region

- In most **engineering** applications, engineering stress and strain is accurate enough, because the cross-sectional area and length of the specimen **do not change substantially** while loads are applied.
- However, in the tensile test, the cross-sectional area and the length of the specimen can **change substantially**.

⇒ The engineering stress is not an accurate measure.

- **A true stress and strain curve has less practical meaning for the engineering application.**

⇒ Because you have to keep tracking the size for each strain, and what the engineers care most is **how strong the materials are** and **when they are going to fail**.

$\sigma_t = P/A_i$; $\varepsilon_t = \Delta l/l_i$ $(l_i = l_0 + \Delta l)$ $\sigma_e = P/A_0$; $\varepsilon_e = \Delta l/l_0$

$$\sigma_t = \frac{P}{A_i} = \frac{Pl_i}{A_i l_i} = \frac{Pl_i}{A_0 l_0} = \frac{P}{A_0}\frac{l_0 + \Delta l}{l_0} = \boxed{\sigma_e(1+\varepsilon_e)}$$

(Assume the volume remains constant)

σ_t: true stress;

σ_e : engineering stress;

P: load

ε_t: true strain;

ε_e: engineering strain;

Δl: increase in length.

$$\varepsilon_t = \int_{l_0}^{l_i} \frac{dl}{l} = \ln\frac{l_i}{l_0} = \ln\frac{l_0 + \Delta l}{l_0}$$

$$= \boxed{\ln(1+\varepsilon_e)}$$

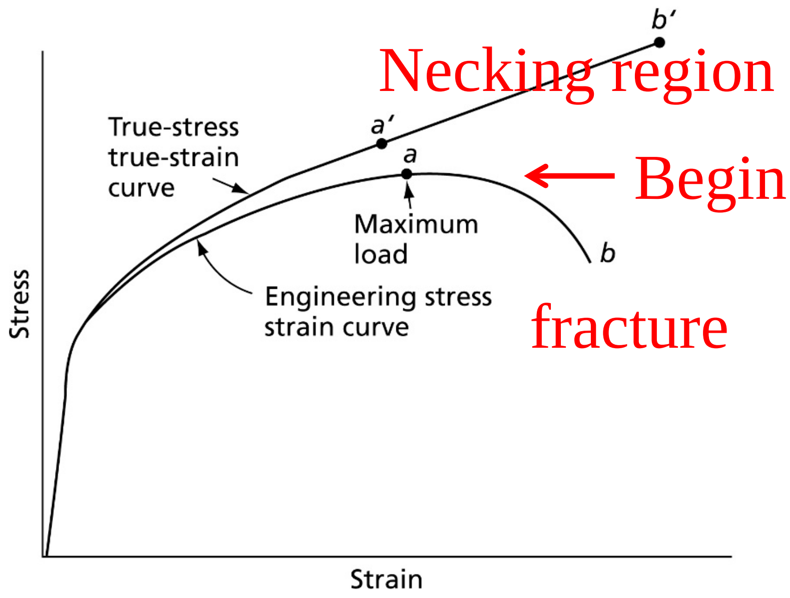
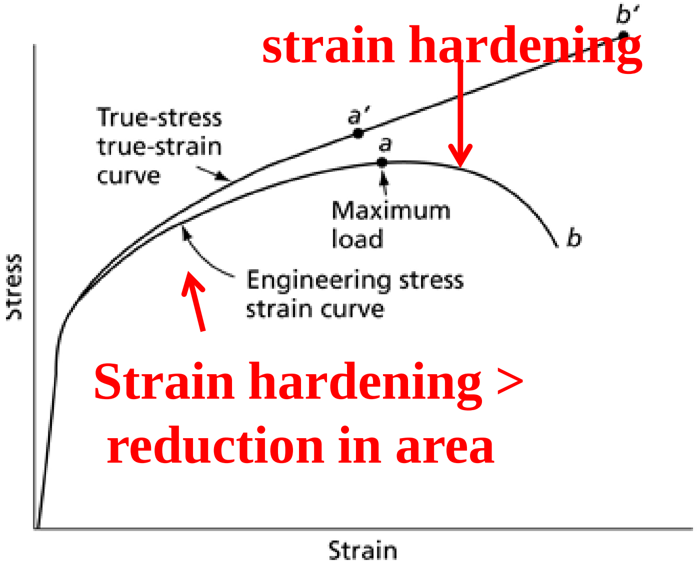


FIG. 5.33 A comparison between an engineering stress-strain curve and the corresponding true-stress and true-strain curve.

- For an engineering stress strain curve, deforms uniformly to approximately the point a, beyond point a
- Necking: a mode of tensile deformation where **relatively large amounts of strain localize** disproportionately in a small region of the material.
- Beyond point b => fracture
- If one considers the triaxiality (simultaneous σ_x, σ_y, σ_z) of the stress, and the neck diameter, one obtains a true stress-strain curve.
- Between a’ and b’ (necking area): approximately a straight line.

5.18 Considere's Criterion

- **Considere's Criterion**: condition for the onset of necking; assumption: necking begins at the point of maximum load.
- Initial **strain hardening** more than compensates for reduction in area. Engineering stress continues to raise with engineering strain.
- Point a is reached where **decrease in area** > increase in strength due to strain hardening.
  - ⇒ Deformation gets **localized**.
  - ⇒ Point a (area) decreases more rapidly than the load due strain hardening. Further elongation occurs with decreasing load.



Reduction in area > strain hardening

Strain hardening > reduction in area

loading

$$P = \sigma_t A_i => dP = \underset{\text{Strain hardening}}{\boxed{A_i d\sigma_t}} + \overset{\text{Area reduction}}{\boxed{\sigma_t dA_i}} = 0 \quad => \quad d\sigma_t = -\sigma_t \frac{dA_i}{A_i}$$

$\boxed{\sigma_t = P/A_i}$

- The **decrease** in load due to decrease in the cross-sectional area is equal to the **increase** in load bearing capacity of the specimen due to strain hardening.
- Another reasonable assumption: volume remains constant

$$dV = d(A_i l_i) = A_i dl_i + l_i dA_i = 0 \quad => \frac{dA_i}{A_i} = -\frac{dl_i}{l_i} = -d\varepsilon_t$$

$$=> \quad \sigma_t = -\frac{d\sigma_t}{\frac{dA_i}{A_i}} = \frac{d\sigma_t}{\frac{dl_i}{l_i}} = \frac{d\sigma_t}{d\varepsilon_t}$$ **: Considere's criterion**

- Necking will occur when the slope of the true-stress true-strain curve ($d\sigma_t/d\varepsilon_t$) is equal to the true stress (σ_t), necking should begin.

5.19 The Relation Between Dislocation Density And The Stress (Experimentally Observed)

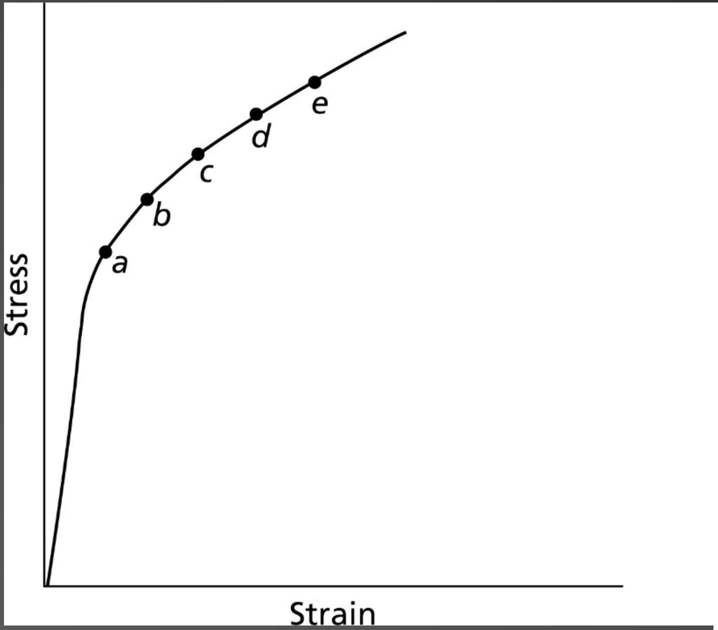
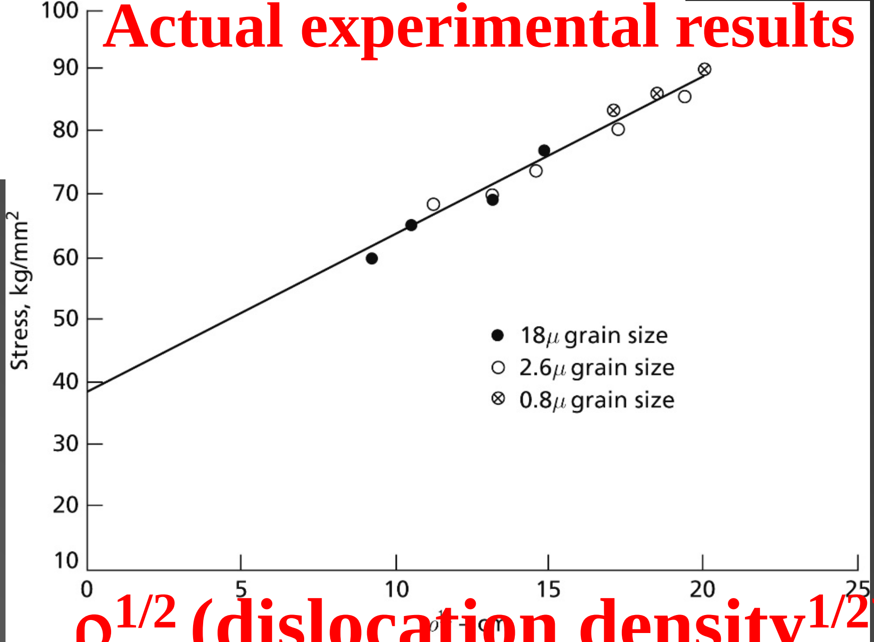


FIG. 5.34 To determine the variation of the dislocation density with strain during a tensile test, a set of tensile specimens are strained to a number of different positions along the stress-strain curve, such as points *a* to *f* in this diagram. These specimens are then sectioned to obtain transmission electron microscope foils



Actual experimental results

$\rho^{1/2}$ (dislocation density$^{1/2}$)

FIG. 5.35 The variation of the flow-stress σ with the square root of the dislocation density $\rho^{1/2}$ for titanium specimens deformed at room temperature, and at a strain rate of 10^{-4} sec^{-1} (After Jones, R. L., and Conrad, H., *TMS-AIME*, **245** 779 [1969].)

- The relation between dislocation density in polycrystalline metals and the stress

$$\sigma = \sigma_0 + k\rho^{1/2}$$

σ : flow stress
ρ : measured dislocation density
σ_0: extrapolated to zero

- Work hardening in metals is directly related to the **build-up of the dislocation density** in the metal.

- Also observed in single crystals, resolved shear stress (τ) is more proper to express the relationship

$$\tau = \tau_0 + k\rho^{1/2}$$

τ : flow stress
ρ : measured dislocation density
τ_0: extrapolated to zero

- **If ρ is zero, the metal could not be deformed.**

⇒ σ_0 and τ_0 are best considered as convenient constants rather than as simple physical properties.

5.20 Taylor's Relation (British physicist & mathematician)

- In 1934, Taylor proposed a **theoretical** relationship between the flow stress and the dislocation density.
  $\Rightarrow$ Equivalent to experimental results.
- Taylor's relation: work hardening results from the **interaction of dislocation**. If ρ is expressed in **#/cm^2**, the flow stress τ is just the stress required to force **two edge dislocations on paralle slip planes of spacing r ($\propto \rho^{-1/2}$) past each other against their elastic interaction**. (ave. dis. between dislocations)
- Same sign: repulsive; opposite sign: attractive. in either case, this interaction must be overcome to continue to glide on their respective slip planes.

$$\tau = \mu\gamma = \mu b/(2\pi r) = \alpha\mu b/r$$

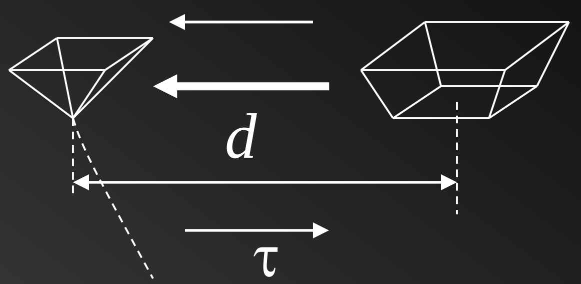
$$\therefore \tau_{\theta z} = \mu r_{\theta z} = \frac{\mu b}{2\pi r}$$

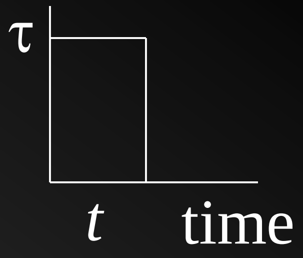
(Screw dislocation)

$$\mathbf{r \propto \rho^{-1/2}}$$

$$\tau = \alpha\mu b \big/ \rho^{-1/2} = \alpha\mu b\rho^{1/2} = k\rho^{1/2} \quad \text{where } k = \alpha\mu b$$

5.21 Dislocation velocity





- Dislocation velocity (v)

$$v = d/t$$

1. From experimental data => Johnston and Gilman found

$$\ln v \propto \ln \tau$$

power law: $v = (\tau/D)^m$

D: the stress yields at v =1 cm s^{-1}; τ: applied shear stress
m: exponent; function of purity, temp. etc.

2. Temperature dependence of v

t_f: time of flight between obstacles

=> $\ln v \propto 1/T$ (phonon effect)

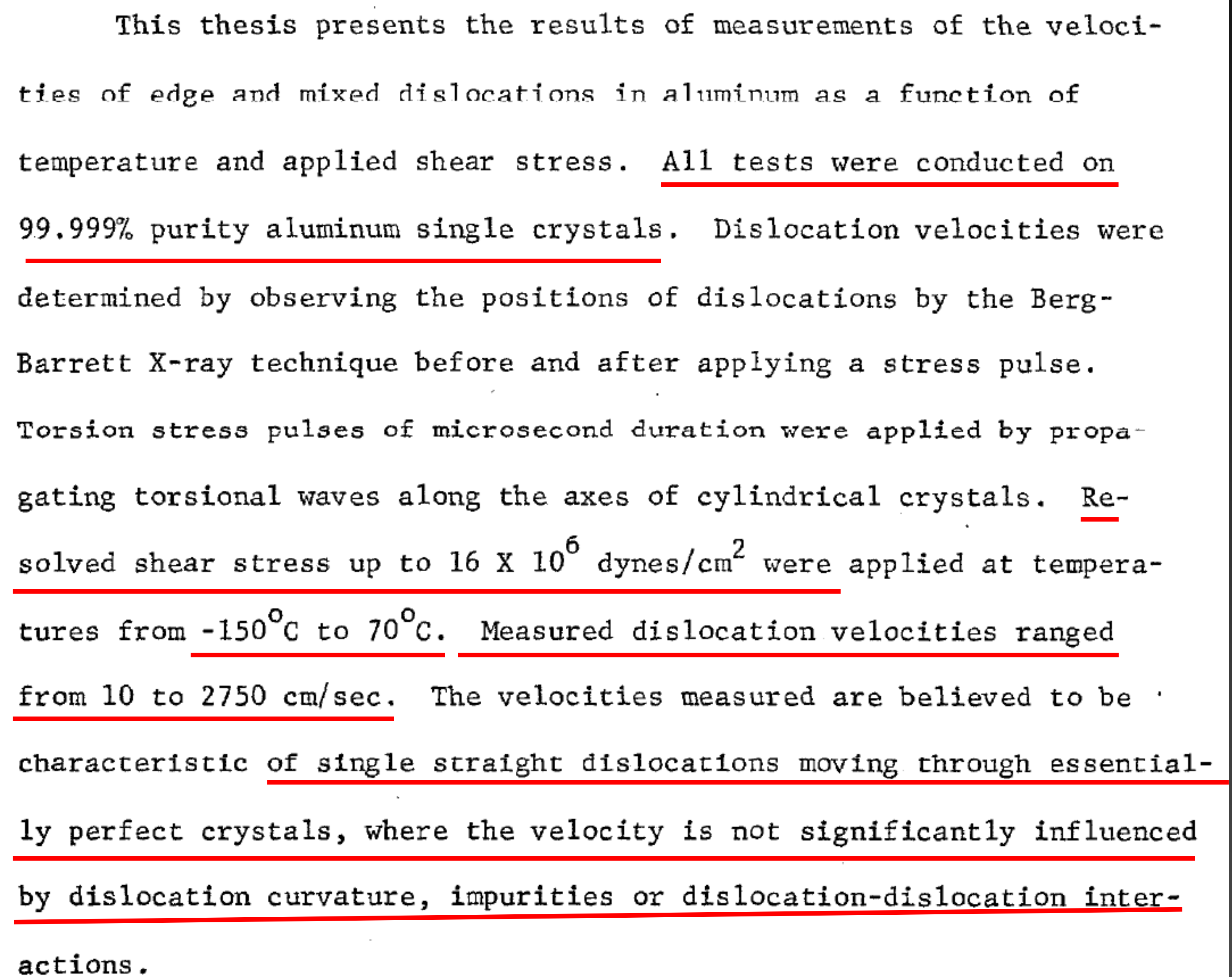
3. Combine 1 & 2: single expression

$$v = \underbrace{f(\sigma)}_{\text{Stress dependence term}}\;\underbrace{e^{-E/kT}}_{\text{Temperature dependence term}}$$

25°C > T > -50°C

E: activation energy
T: absolute temperature
k: Boltzmann’s constant

The Mobility of Dislocations in High Purity Aluminum



This thesis presents the results of measurements of the velocities of edge and mixed dislocations in aluminum as a function of temperature and applied shear stress. All tests were conducted on 99.999% purity aluminum single crystals. Dislocation velocities were determined by observing the positions of dislocations by the Berg-Barrett X-ray technique before and after applying a stress pulse. Torsion stress pulses of microsecond duration were applied by propagating torsional waves along the axes of cylindrical crystals. Resolved shear stress up to 16 X 10^6 dynes/cm^2 were applied at temperatures from -150°C to 70°C. Measured dislocation velocities ranged from 10 to 2750 cm/sec. The velocities measured are believed to be characteristic of single straight dislocations moving through essentially perfect crystals, where the velocity is not significantly influenced by dislocation curvature, impurities or dislocation-dislocation interactions.

J. A. Gorman, Ph.D. Thesis, Cal. Tech. 1968

Intrinsic Mobility of a Dissociated Dislocation in Silicon

Physical Review Letters 84, 3346 (2000).

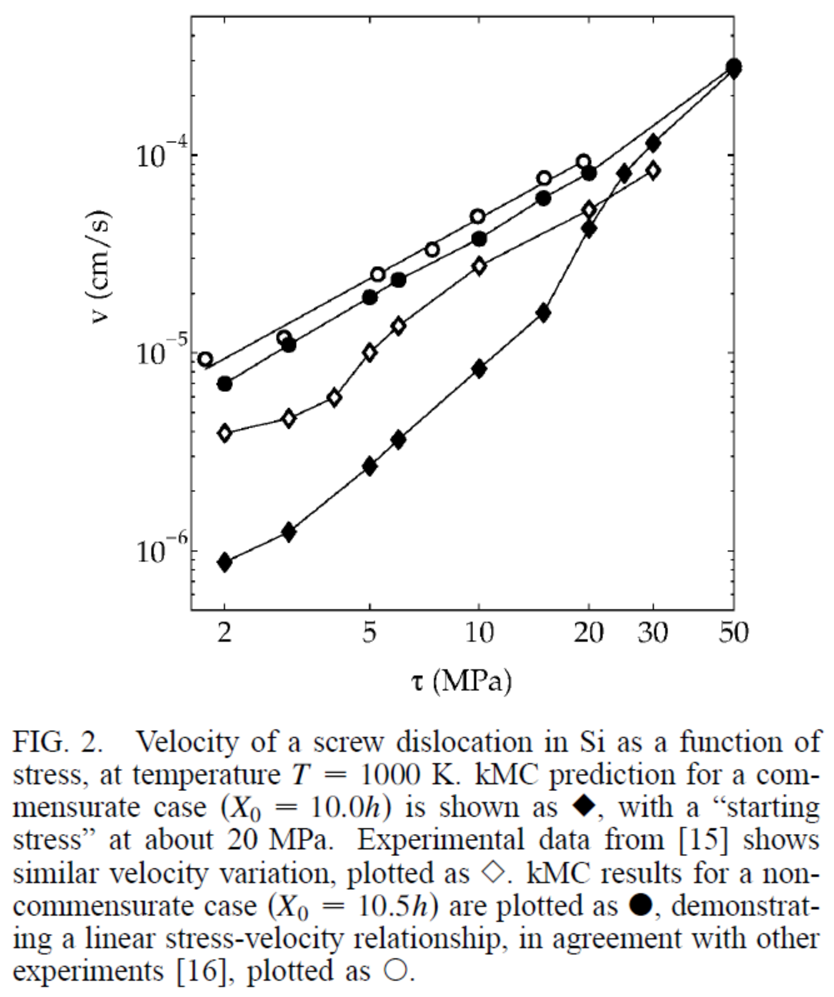
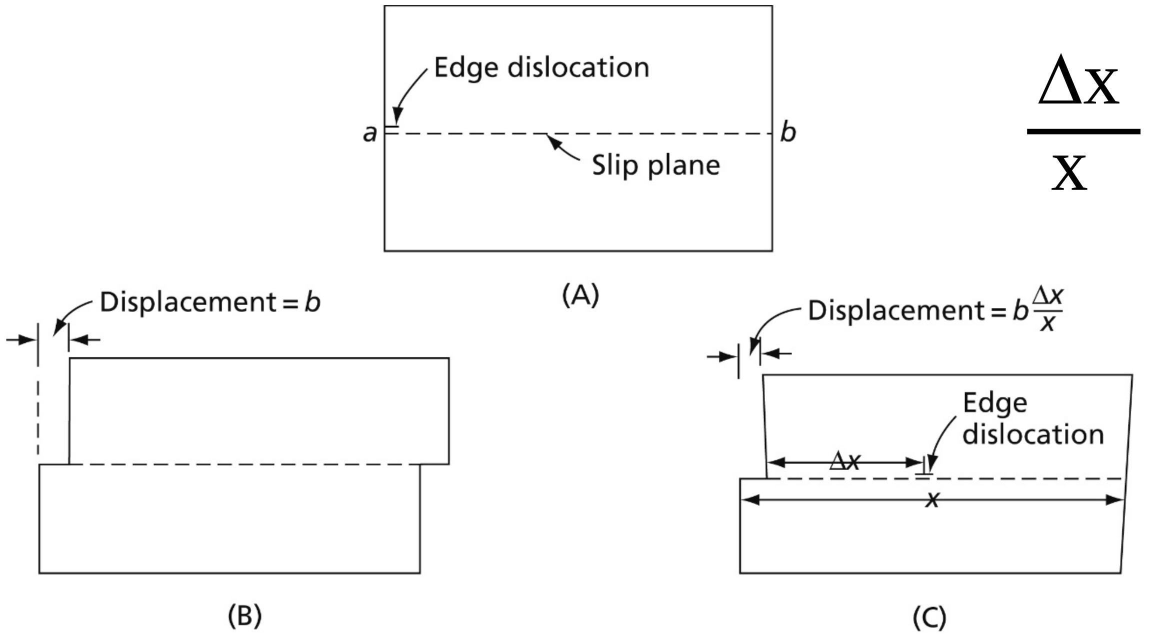


FIG. 2. Velocity of a screw dislocation in Si as a function of stress, at temperature $T = 1000$ K. kMC prediction for a commensurate case ($X_0 = 10.0h$) is shown as ◆, with a “starting stress” at about 20 MPa. Experimental data from [15] shows similar velocity variation, plotted as ◇. kMC results for a noncommensurate case ($X_0 = 10.5h$) are plotted as ●, demonstrating a linear stress-velocity relationship, in agreement with other experiments [16], plotted as ○.

The speed of sound

Air	343.2 m/s	→	1200 km/hr
water	1,484 m/s		
Iron	5,120 m/s		

5.23 The Orowan Equation (Strain Rate)



$$\frac{\Delta x}{x} = \frac{\text{displacement}}{b}$$

FIG. 5.36 The displacement of the two halves of a crystal is in proportion to the distance that the dislocation moves on its slip plane

- Relation between the velocity (*v*) of the dislocations and the applied strain rate ($\dot{\varepsilon}$).

in (c): Shear by an amount = $\dfrac{b\Delta x}{x} = \dfrac{b\Delta A}{A}$

=> Shear strain $\Delta\gamma \propto (b\Delta x/x)/x$

$\Delta\gamma \propto (b\Delta x/x)/x = \rho b\Delta x$ (because $(1/x^2) = \rho$)

ρ: the dislocation density

$\boxed{r \propto \rho^{-1/2}}$

Shear strain rate: $$\frac{\Delta\gamma}{\Delta t} = \dot{\gamma} = \frac{\rho b \Delta x}{\Delta t} = \rho b \bar{v}$$

$\bar{v}$: the average dislocation velocity

- For the tensile strain rate ($\dot{\varepsilon}$) in a polycrystalline metal: a reasonable assumption:

Tensile strain rate: $$\dot{\varepsilon} = \frac{1}{2}\dot{\gamma} = \frac{1}{2}\rho b \bar{v}$$

$\uparrow$

Schmid orientation factor

5.22 The Discontinuous Nature of Dislocation Movement

- In real crystal, there are a lot of obstacles in lattice
  => the movement of a dislocation is not smooth and continuous, but rather it occurs in steps.
  => Moves rapidly for a short distance; it stops and waits at an obstacle while eventually it passes; it moves rapidly again to the next obstacle.

l: average distance between obstacles

$$v = \frac{l}{t_f + t_w} = \frac{l}{t_w}$$

t_f: time of flight between obstacles

t_w: average time the dislocation waits at an obstacle for the thermal energy to become large enough to allow the dislocation **to penetrate the obstacle**.

$$t_f << t_w$$

⇒ Thermal vibrations aid the applied stress to overcome these obstacles to dislocation motion

⇒ **Temperature dependent**

Creep deformation

- A stress greater than or equal to its yield stress
  => the material deforms plastically.
- The stress is below the yield level
  => **in principle it should only deform elastically.**
- However, **provided the temperature is relatively high**, **plastic deformation can occur** even when the stress is lower than the yield stress.
  => This deformation is time-dependent and is known as **creep**.

https://www.doitpoms.ac.uk/tlplib/creep/intro.php

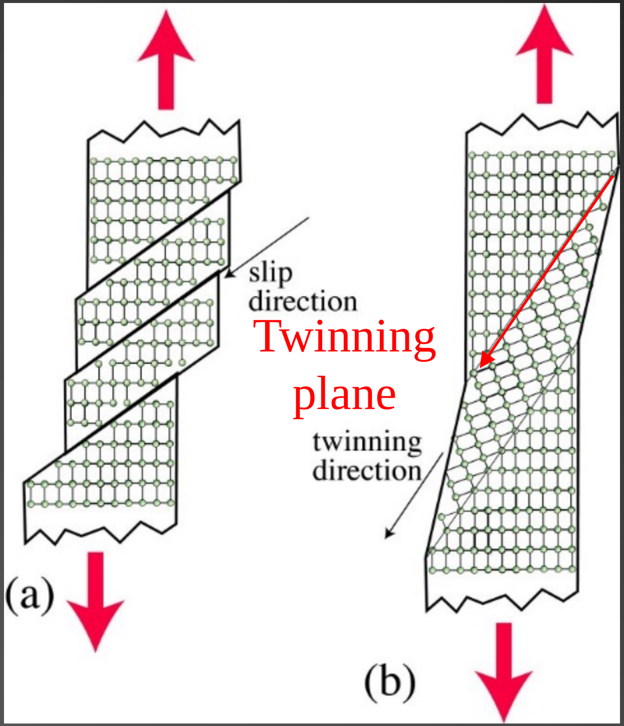
- A general rule: creep starts to become significant when the homologous temperature is **greater than 0.4 T_m**.
- Most metals **do not suffer from creep at room temp.** because they have much higher melting points than solder.
- However, creep can still be a major concern when designing metallic components that have to function at high temp.
- An example: **the design of turbine blades** for use in jet engines.
  - ⇒ The blades in these engines can be exposed to hot gases at up to about 1400°C. They are also under stress (the high centrifugal forces).
  - ⇒ These blades must withstand this environment without excessive creep.

Deformation Twinning

Plastic deformation (shear): two primary mechanisms:

(1) slip: a slower and more chaotic process where multiple atomic planes slide over one another.

(2) twinning: one part of the crystal shears with respect to the other along a preferred plane.



https://www.doitpoms.ac.uk/ldplib/shape_memory/background.php
https://physics.aps.org/articles/v9/19

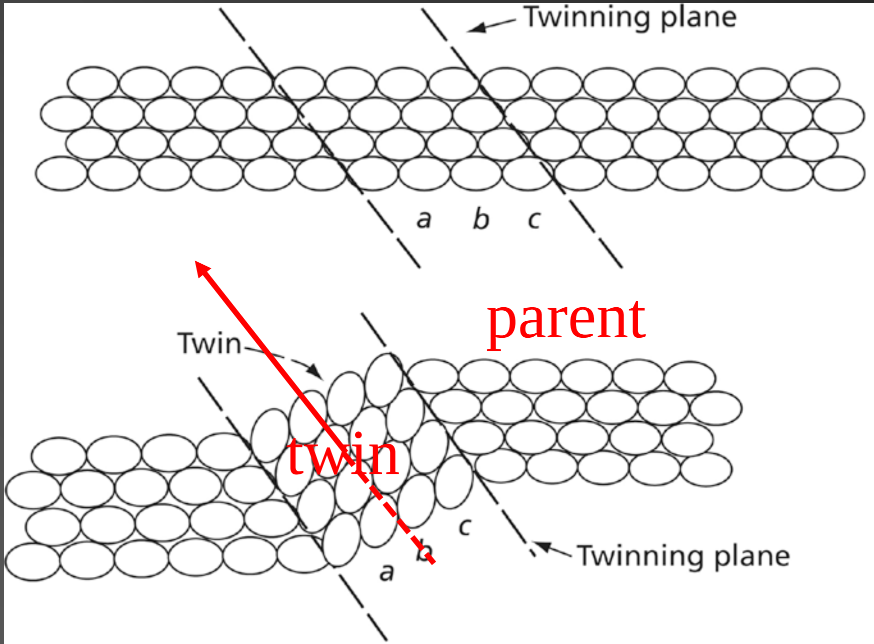
- No change in crystal structure, merely a reorientation of the lattice.

- The shear associated with deformation twinning is uniformly distributed over a volume rather than localized on a discrete number of slip planes.

- The atoms move only a fraction of an interatomic spacing relative to each other. The total shear deformation due to twinning is small.

$\Rightarrow$ Slip is a much more important primary mode of plastic deformation.

DOI: 10.1038/srep15331



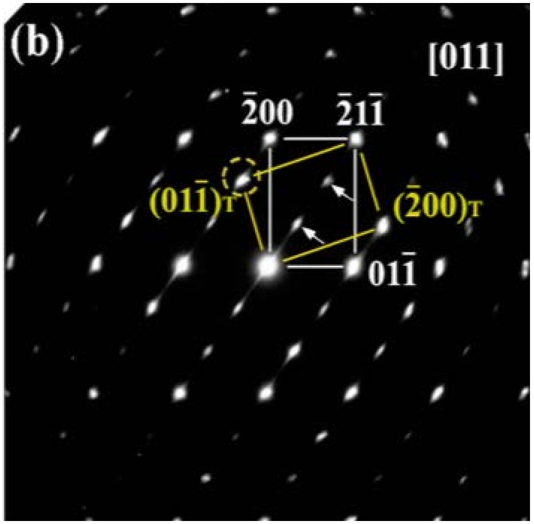


FIG. 17.3 Schematic representation showing how a twin may be produced by a simple movement of atoms

- The twin is formed by the rotation of each atom in the deformed area about an axis through its center and perpendicular to the plane of the paper.

- The lattice of the twin is a mirror image of the parent lattice. The lattices of twin and parent are symmetrically oriented across a symmetry plane called the twin plane.